# WAVE DRAG AND WAVE THRUST PHENOMENA

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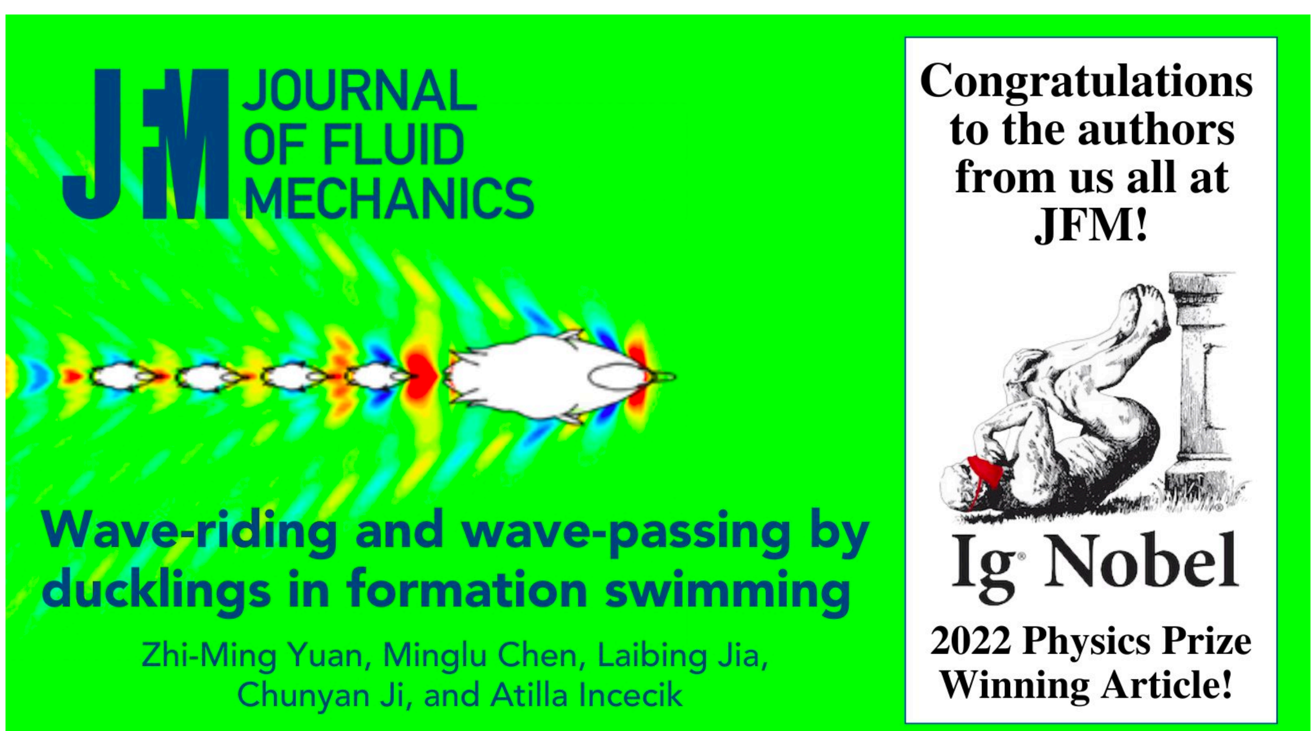
# DUCKS IN A ROW

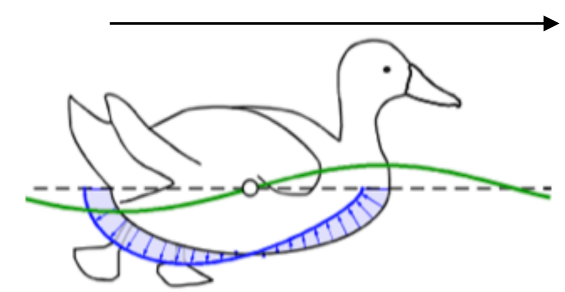
*https://www.youtube.com/user/RobinEAdams*



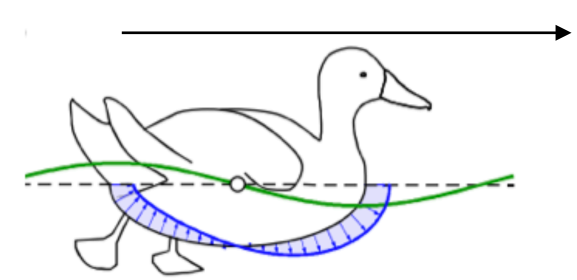

WAVE DRAG (WAKE)

WAVE THRUST (SURFING WAVES)

Poor positioning:
Swimming **against** surface gradients



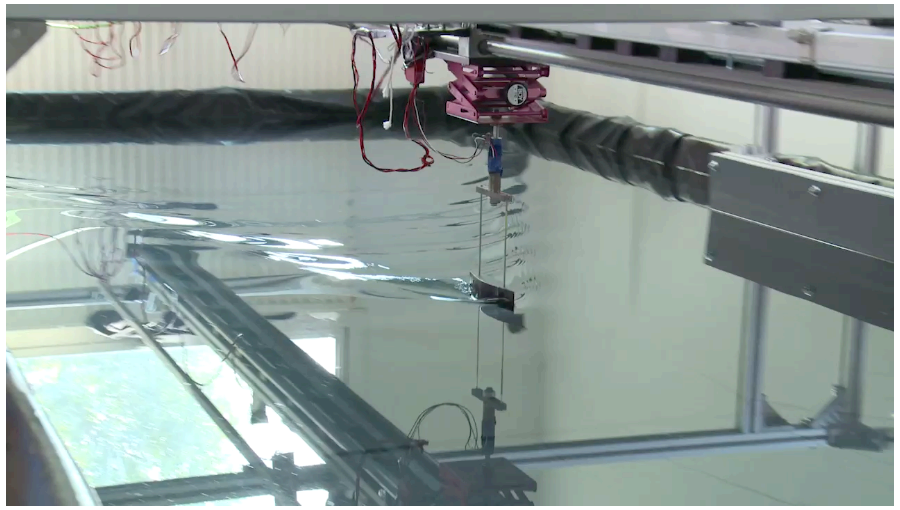
Good positioning:
Swimming **with** surface gradients



# LECTURE OUTLINE

## WAVE DRAG PHENOMENA

1. SHAPE OF OBJECT
2. DEPTH OF WATER

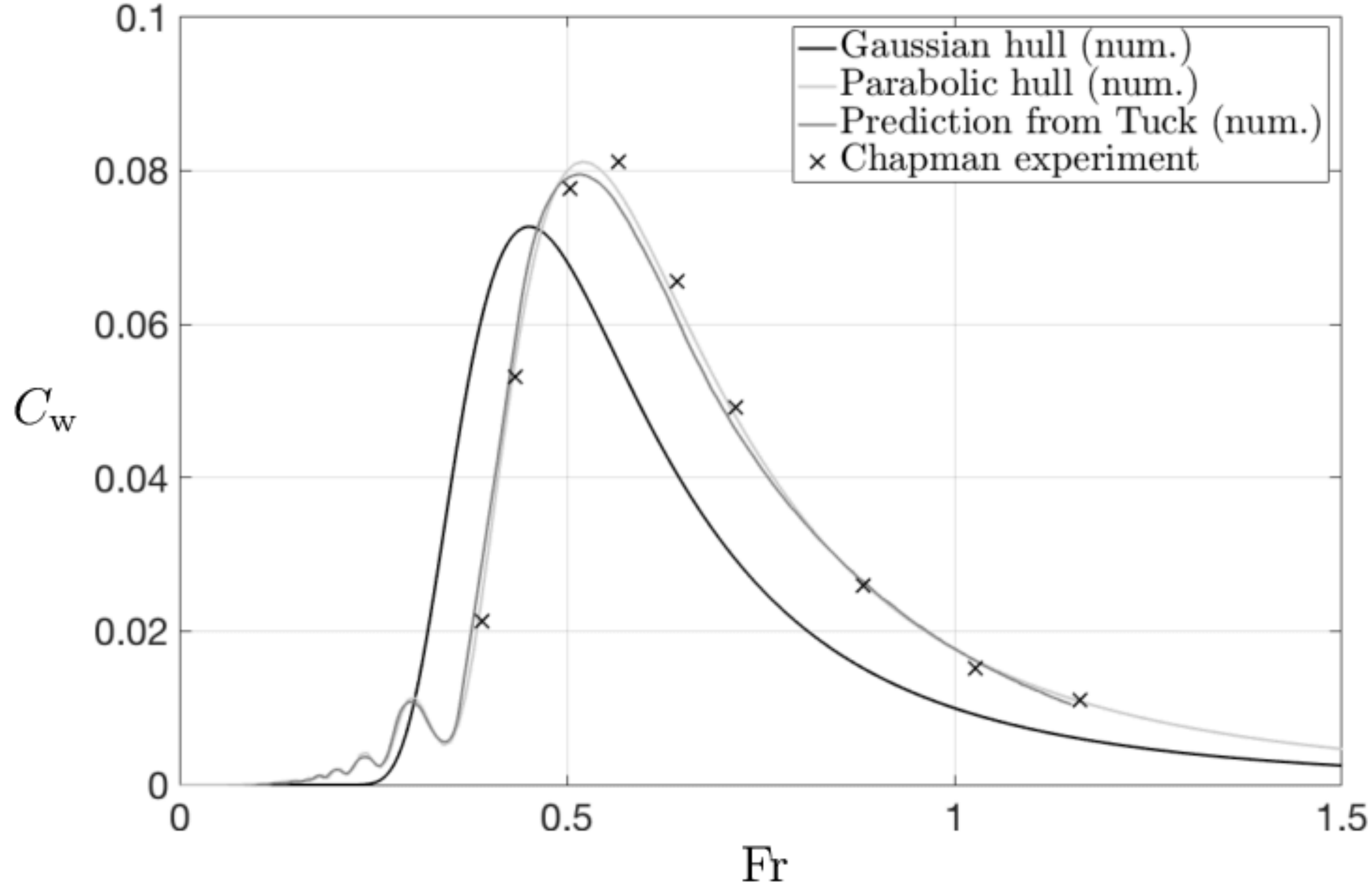


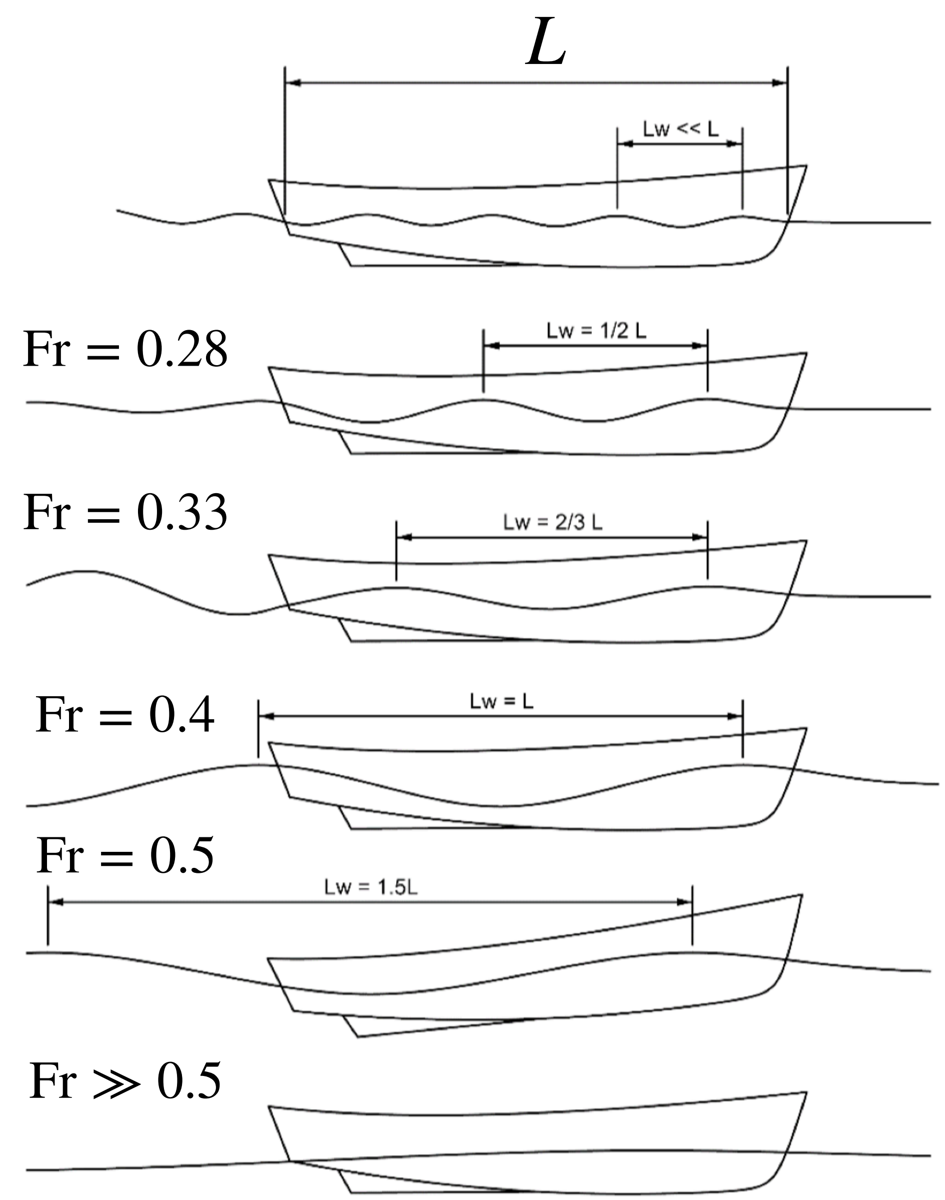
## WAVE THRUST PHENOMENA

1. GUNWALE BOBBING
2. SURFERBOT

# WAVE DRAG IN DEEP WATER

*Boucher et al. PRF (2018)*



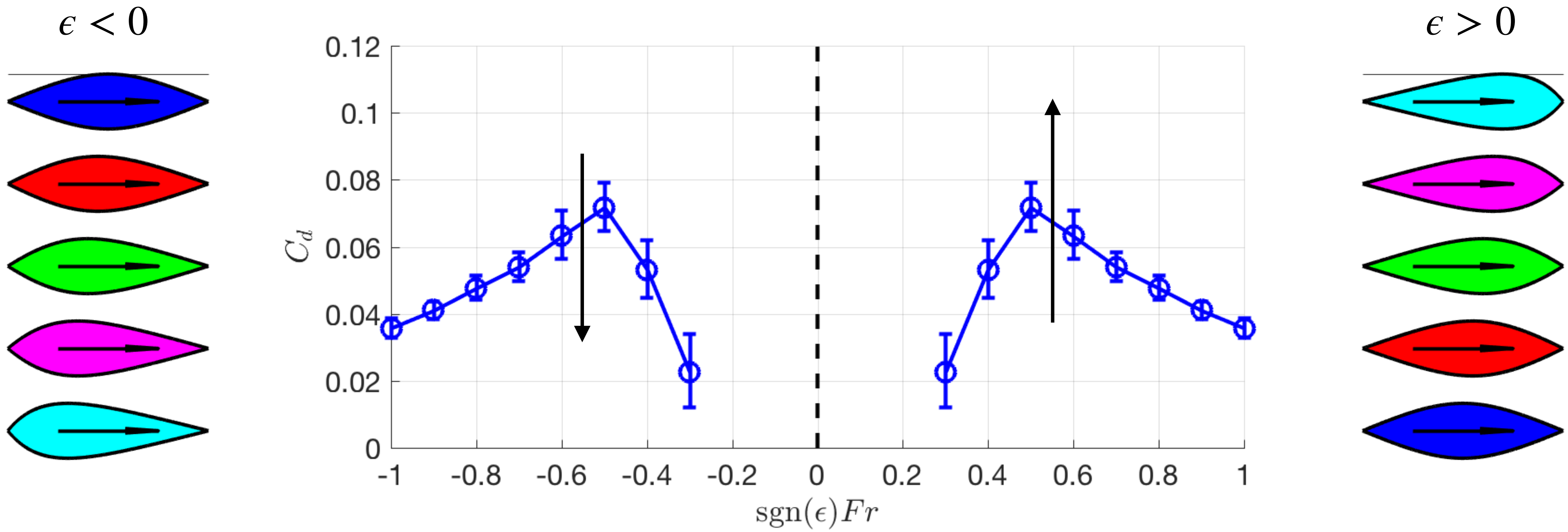




$$C_w = \mathrm{Drag}/\rho U^2 L^2$$

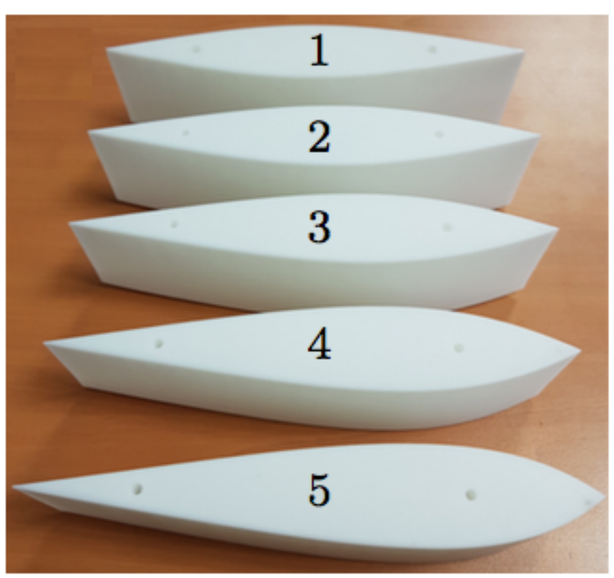
$$\mathrm{Fr} = U/\sqrt{gL}$$

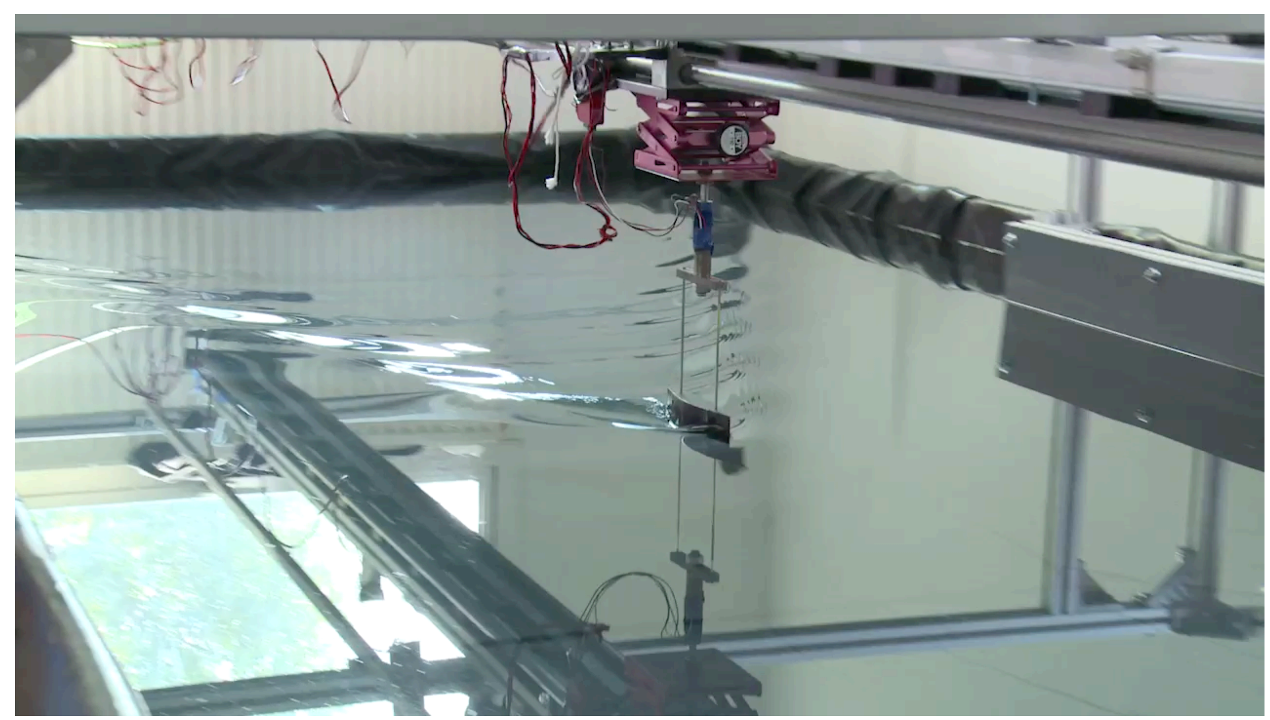
Resonance between boat length and wavelength

# 1. EFFECT OF SHAPE ASYMMETRY



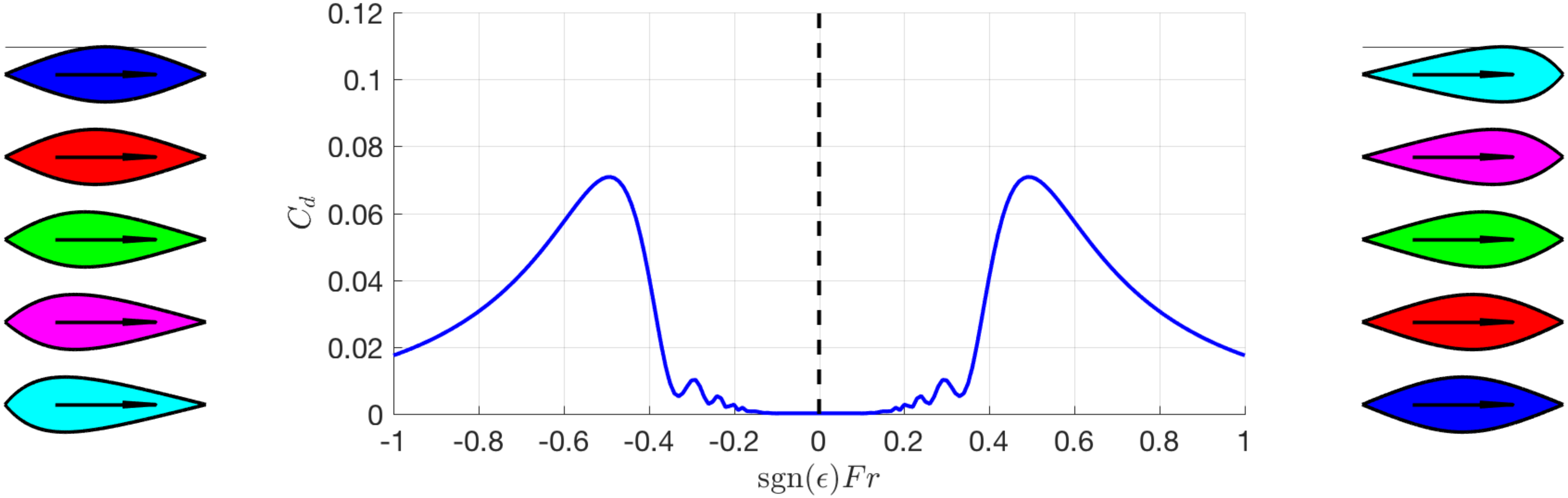

Parameterised family of curves



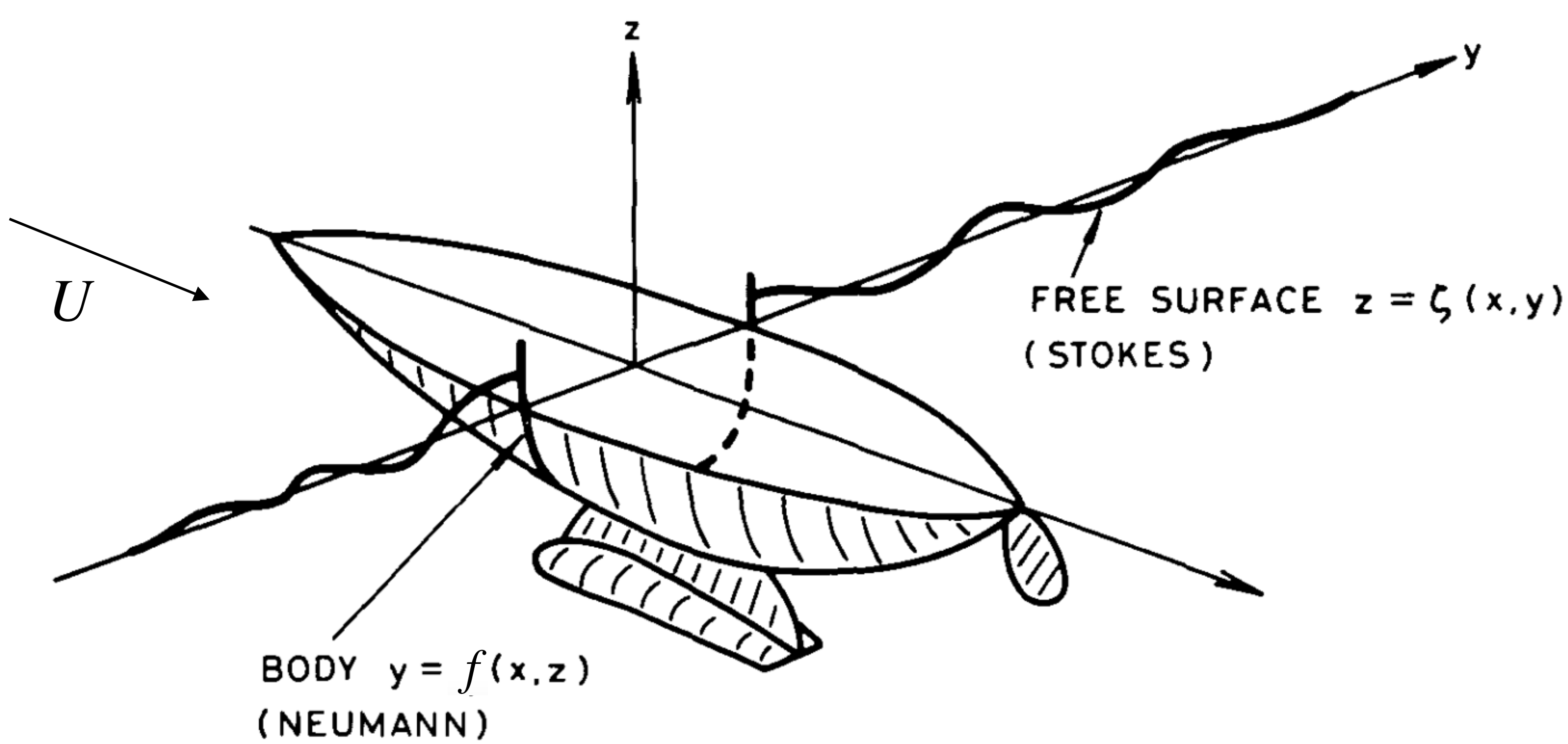



Tow tank with a force sensor

# MICHELL'S THEORY FOR THE WAVE DRAG




*J.H. Michell (1898)*




Based on "Slender body theory"

Fourier transform of $f(x,z)$

Cannot capture asymmetry!

# SYMMETRY PARADOX

EULER EQUATIONS:

$$\nabla \cdot \boldsymbol{u} = 0,$$
$$\rho(\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla p - \rho g \hat{\boldsymbol{k}}$$

BOUNDARY CONDITIONS:

$$v = \pm u f'(x), \quad \text{on } y = \pm f(x),$$

$$w = u\zeta_x + v\zeta_y, \quad \text{on } z = \zeta(x, y),$$
$$p = p_{atm}, \quad \text{on } z = \zeta(x, y),$$

$$\boldsymbol{u} \to (-U, 0, 0), \quad x, y, z \to \pm\infty,$$

APPLY THE TRANSFORMATION:

$$U \to -U, \quad \mathbf{u} \to -\mathbf{u}$$

# SYMMETRY PARADOX

EULER EQUATIONS:

$$\nabla \cdot \boldsymbol{u} = 0,$$

$$\rho\frac{\partial \mathbf{u}}{\partial t} + \rho(\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla p - \rho g\hat{\boldsymbol{k}} + \mu \nabla^2 \mathbf{u}$$

BOUNDARY CONDITIONS:

$$v = \pm u f'(x), \quad \text{on} \;\; y = \pm f(x),$$

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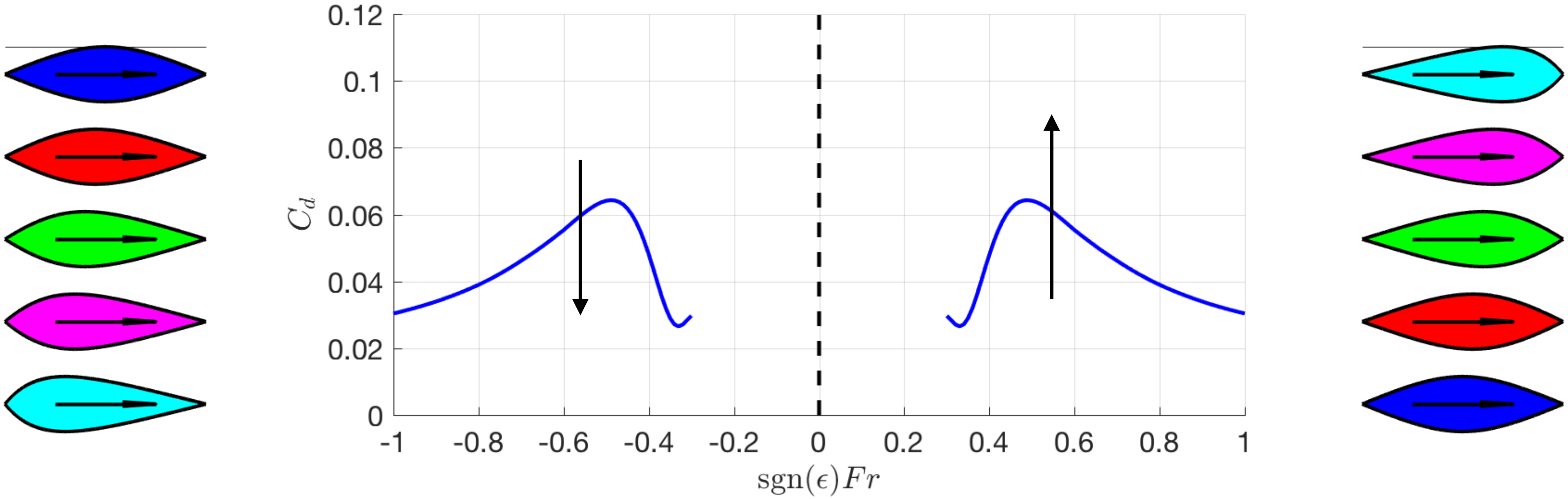
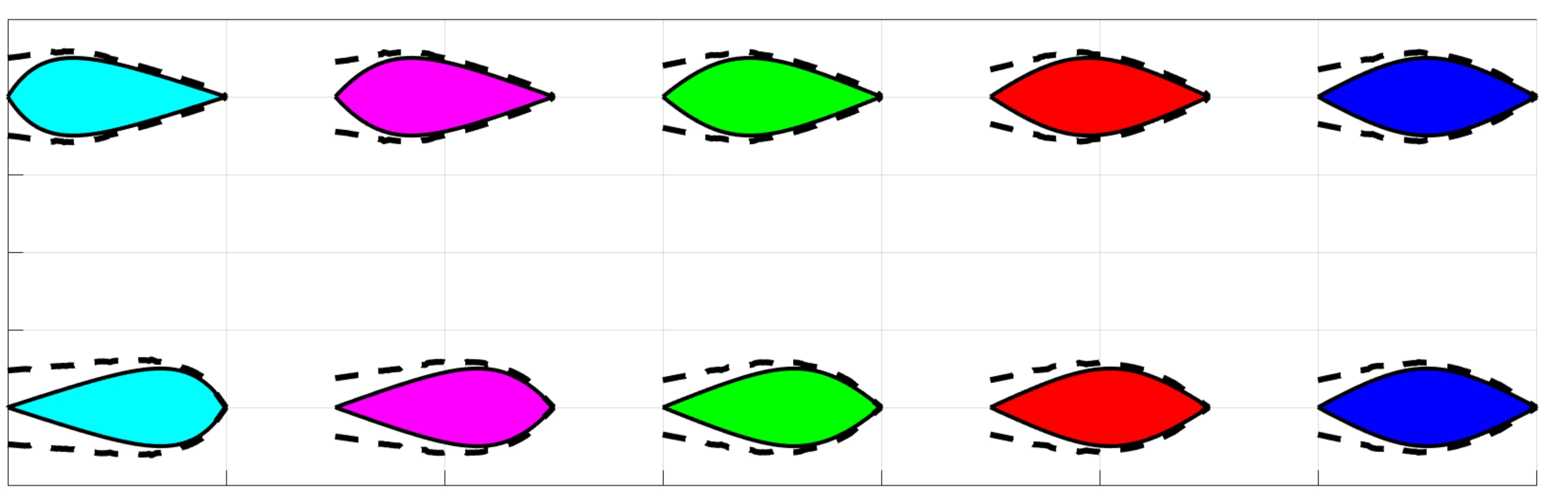
# MODIFIED MICHELL'S THEORY



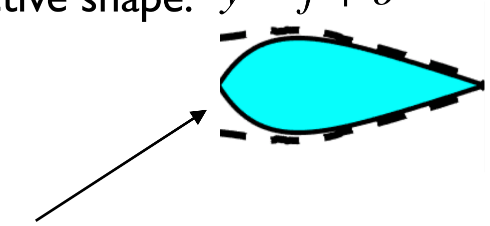

## Effective shape: Hull + Boundary layer



Numerical calculations of boundary layer (OpenFoam)

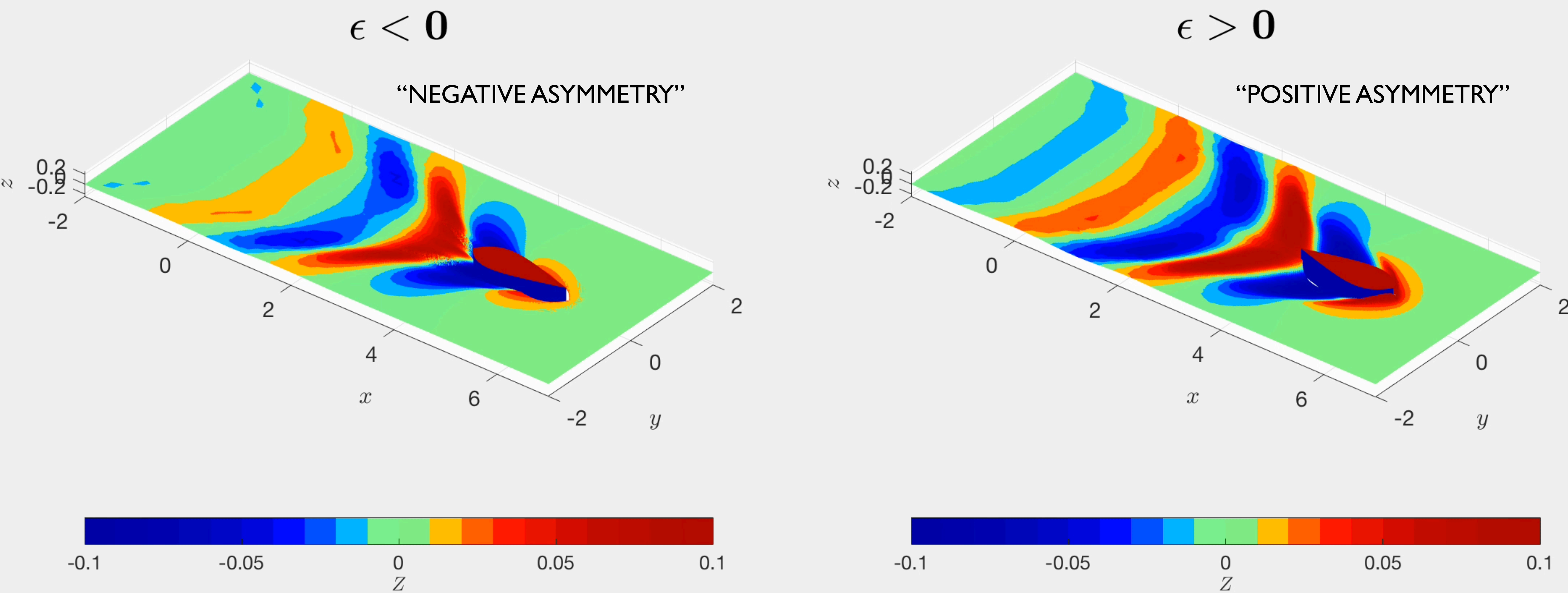
Potential flow $\nabla^2 \phi = 0$

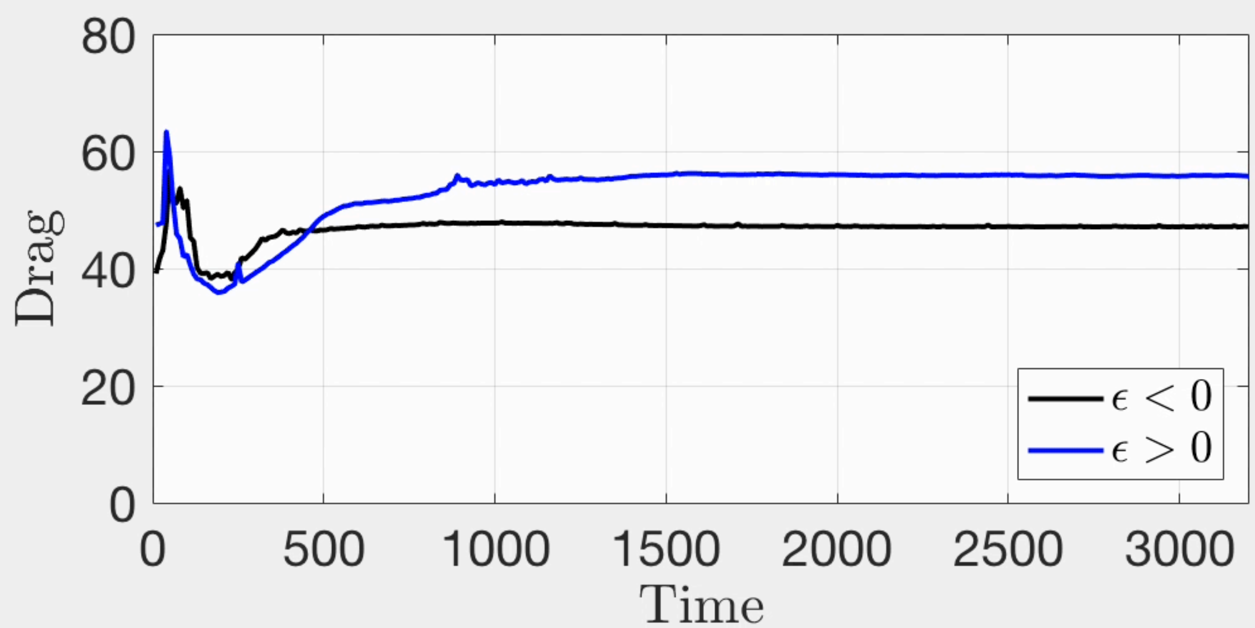
Effective shape: $y = f + \delta$



Turbulent boundary layer

# OPENFOAM SIMULATIONS




*GPB et al., JFM (2019)*

# 2. EFFECT OF WATER DEPTH

*Rodrigo de Freitas Lagoon, Rio de Janeiro, Brazil*

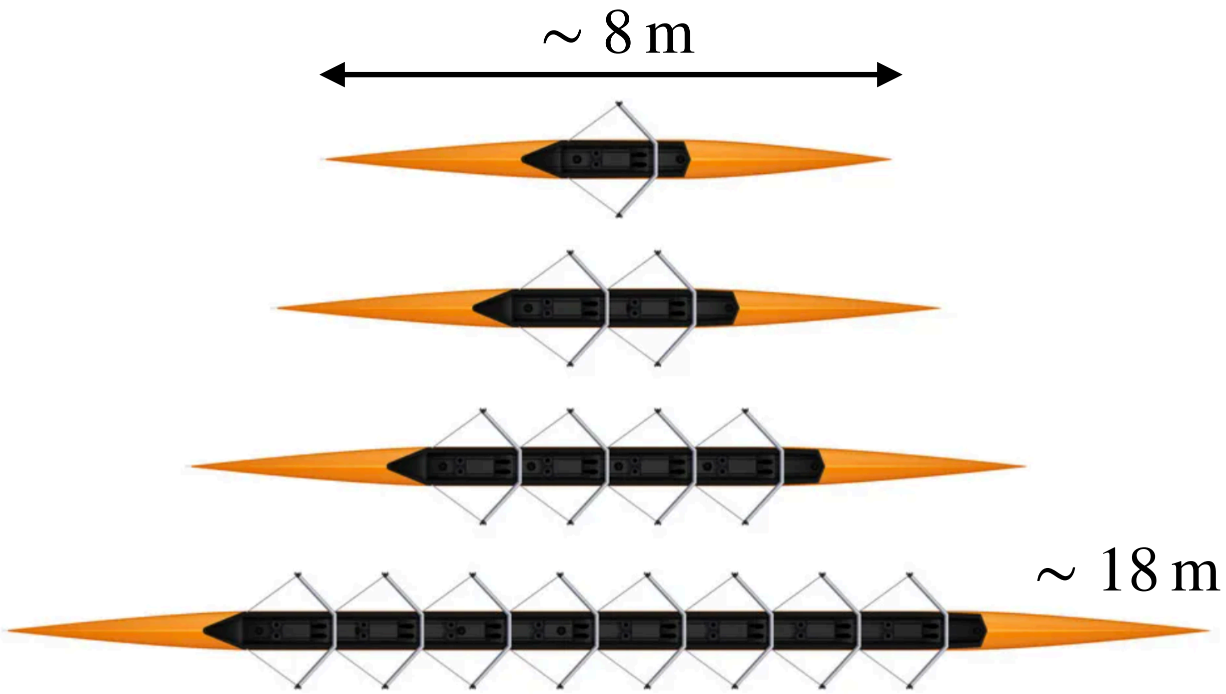
Mean depth: 2.8 m, Maximum depth: 4.3 m

*Lake Bled, Slovenia*

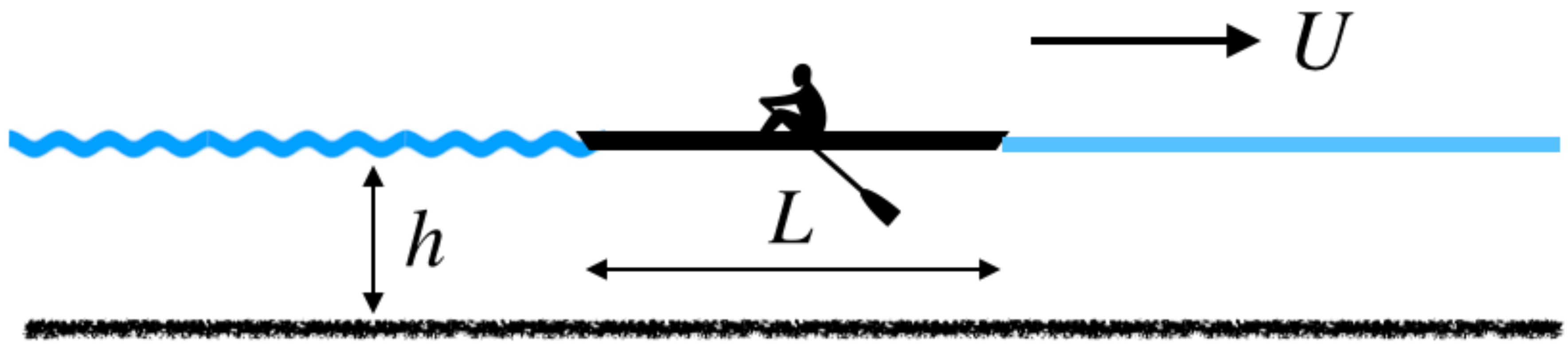
Maximum depth: 30 m, with a small island

*Sea forest waterway, Koto, Japan*

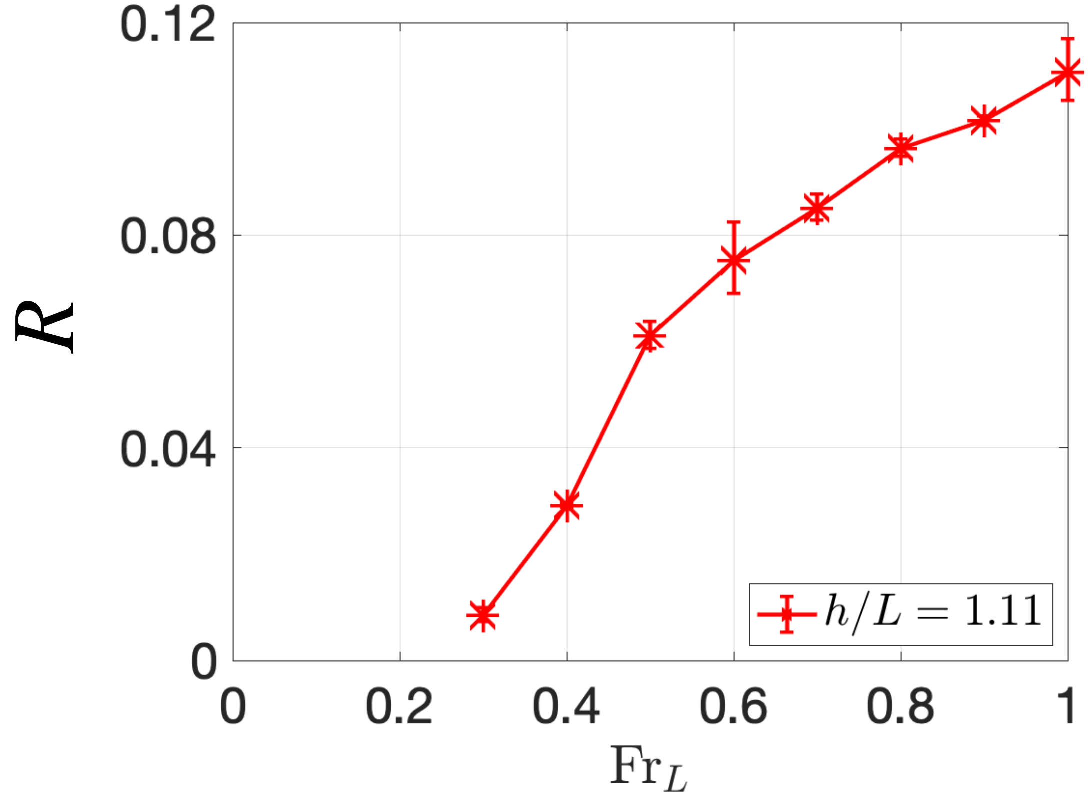
Consistent depth: 6m



Rafid Bendimerad




EXPERIMENTS



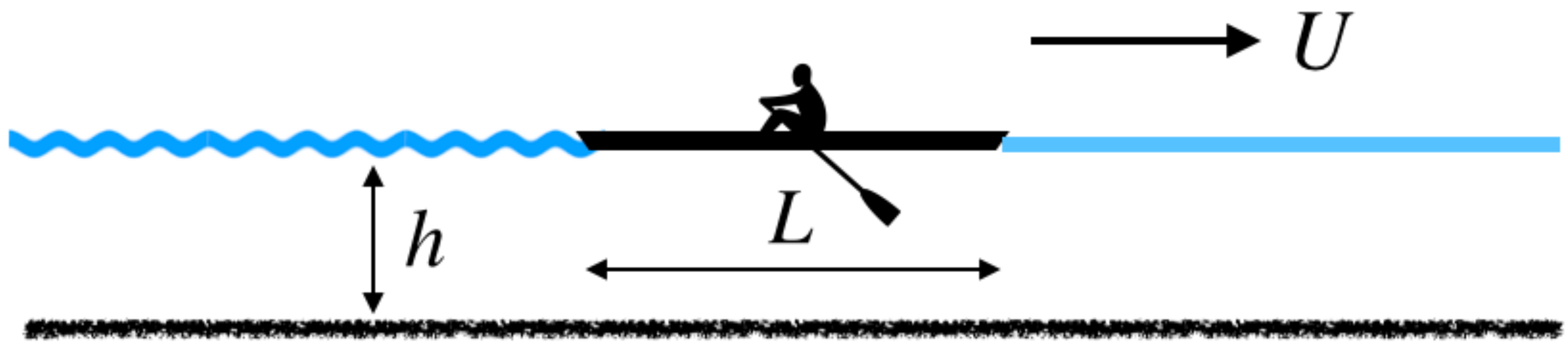

$$\mathrm{Fr}_L = U/\sqrt{gL}$$

$$\mathrm{Fr}_h = U/\sqrt{gh}$$

Normalisation:

$$R = \mathrm{Drag}/\rho g L^3$$

Rafid Bendimerad

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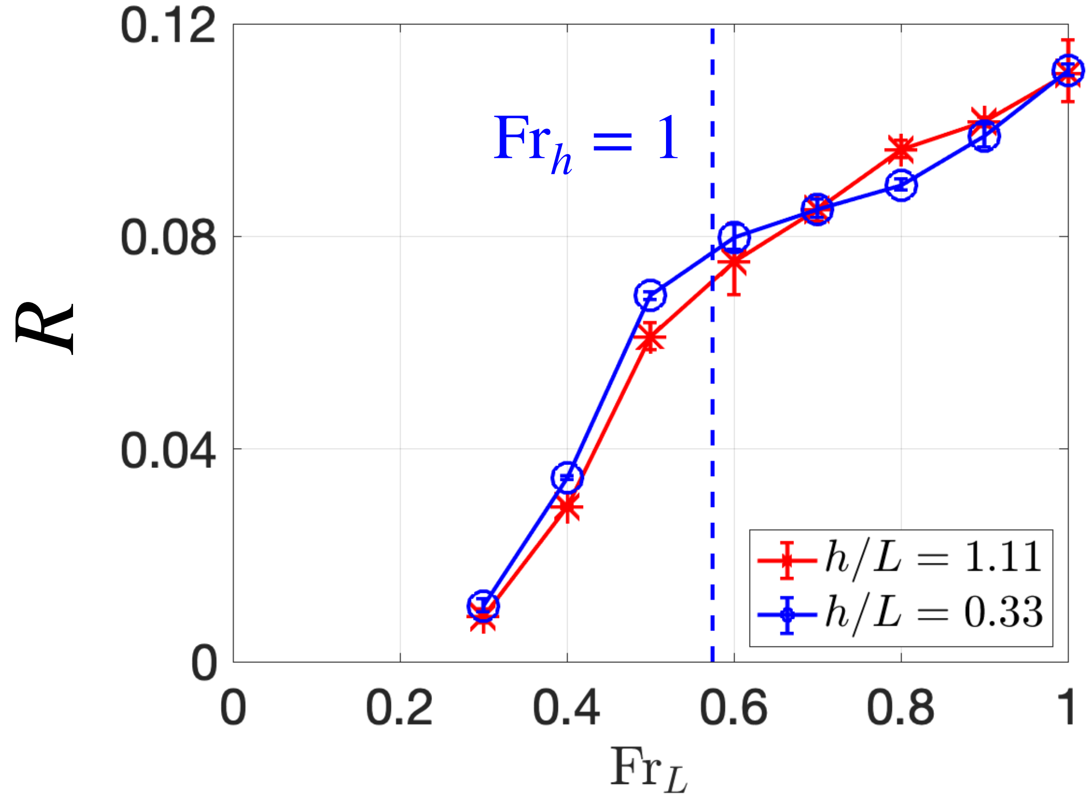


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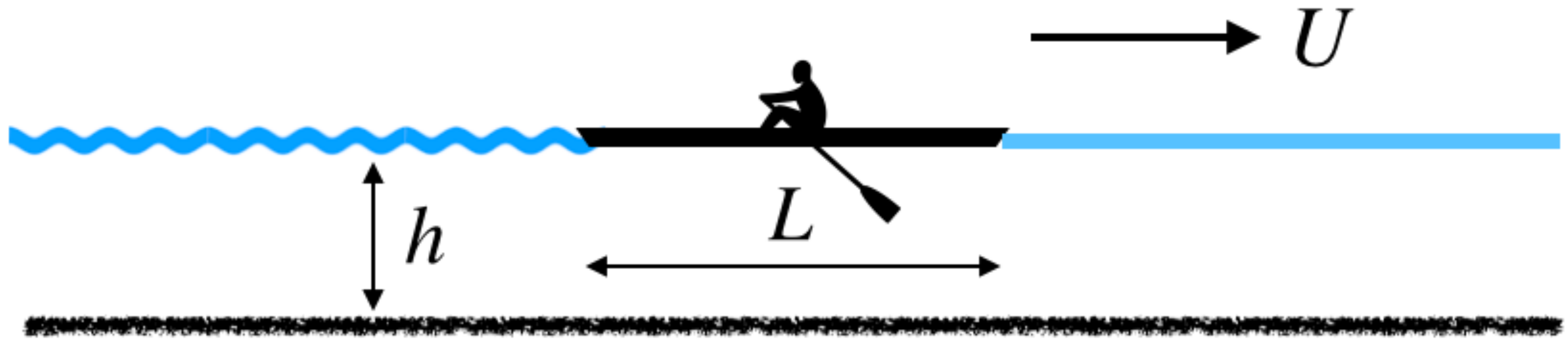
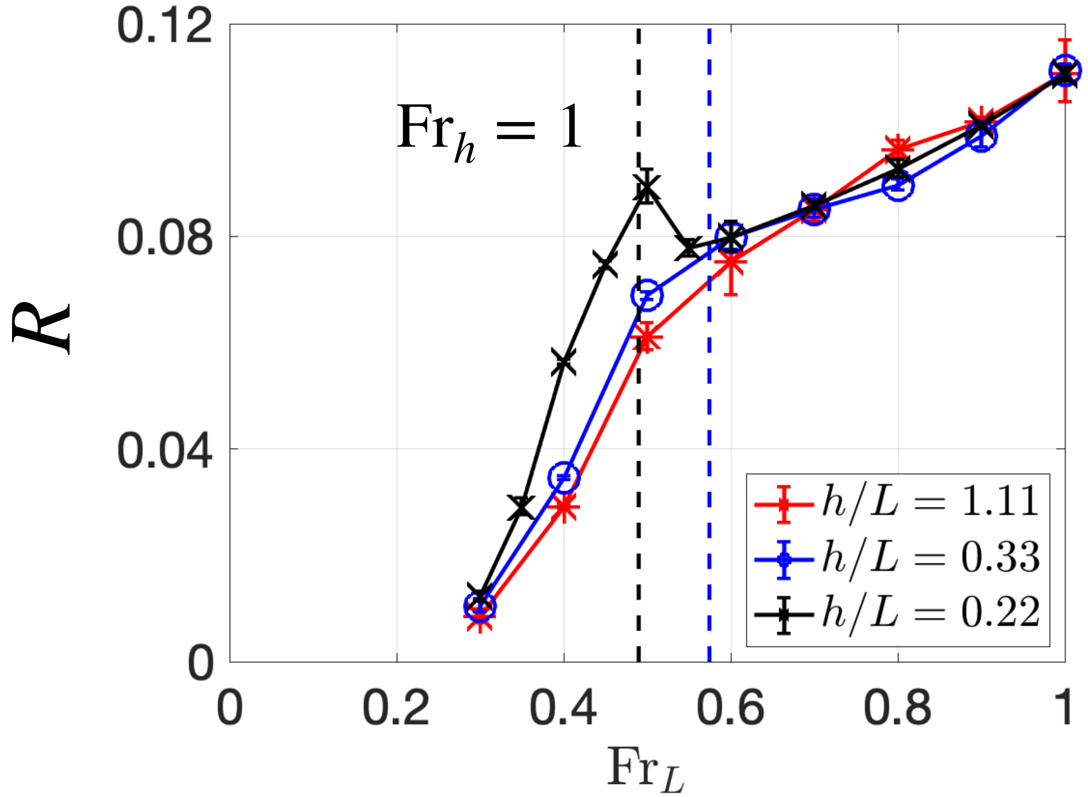
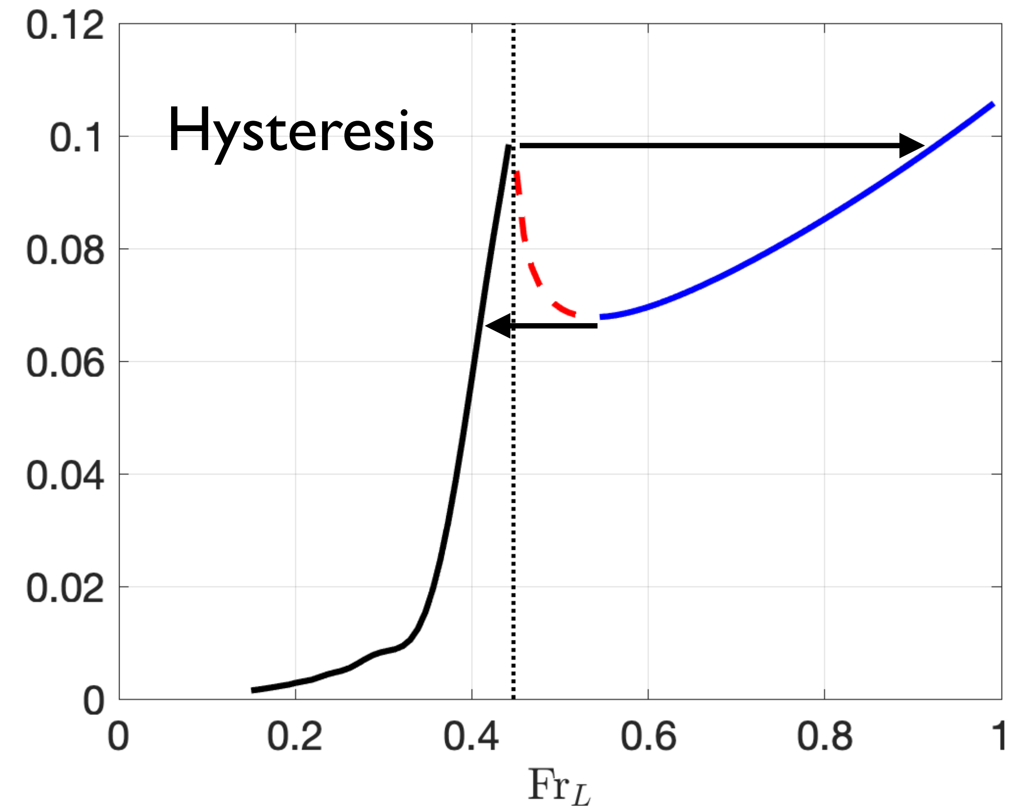



EXPERIMENTS




LINEAR THEORY

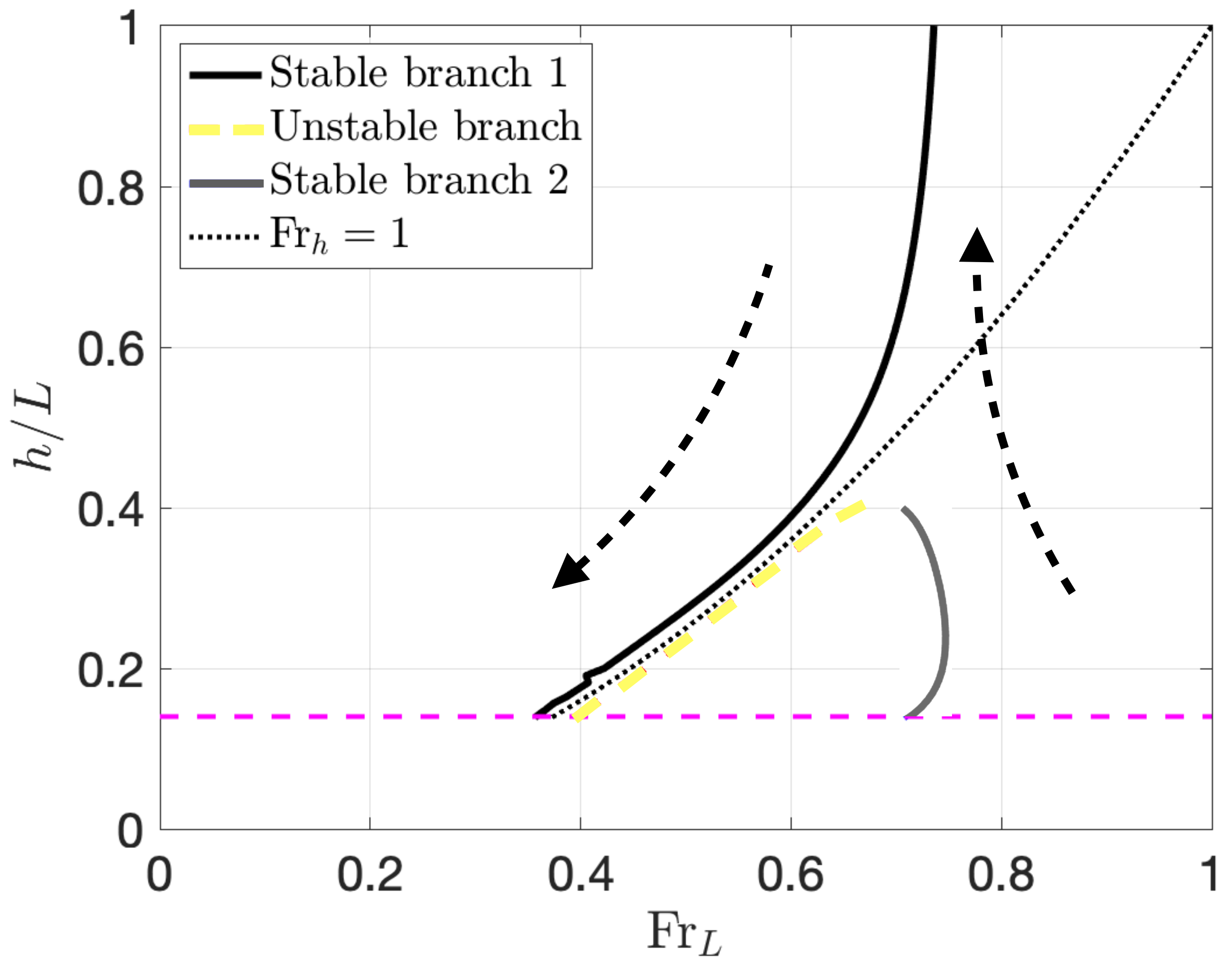



$$\mathrm{Fr}_L = U/\sqrt{gL}$$

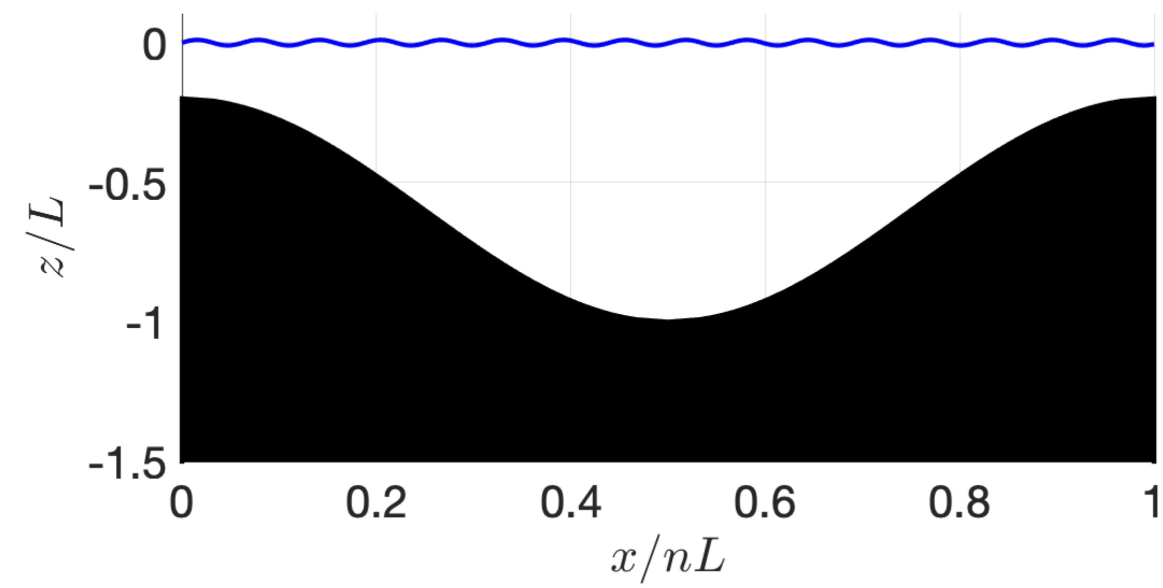
$$\mathrm{Fr}_h = U/\sqrt{gh}$$

Normalisation:

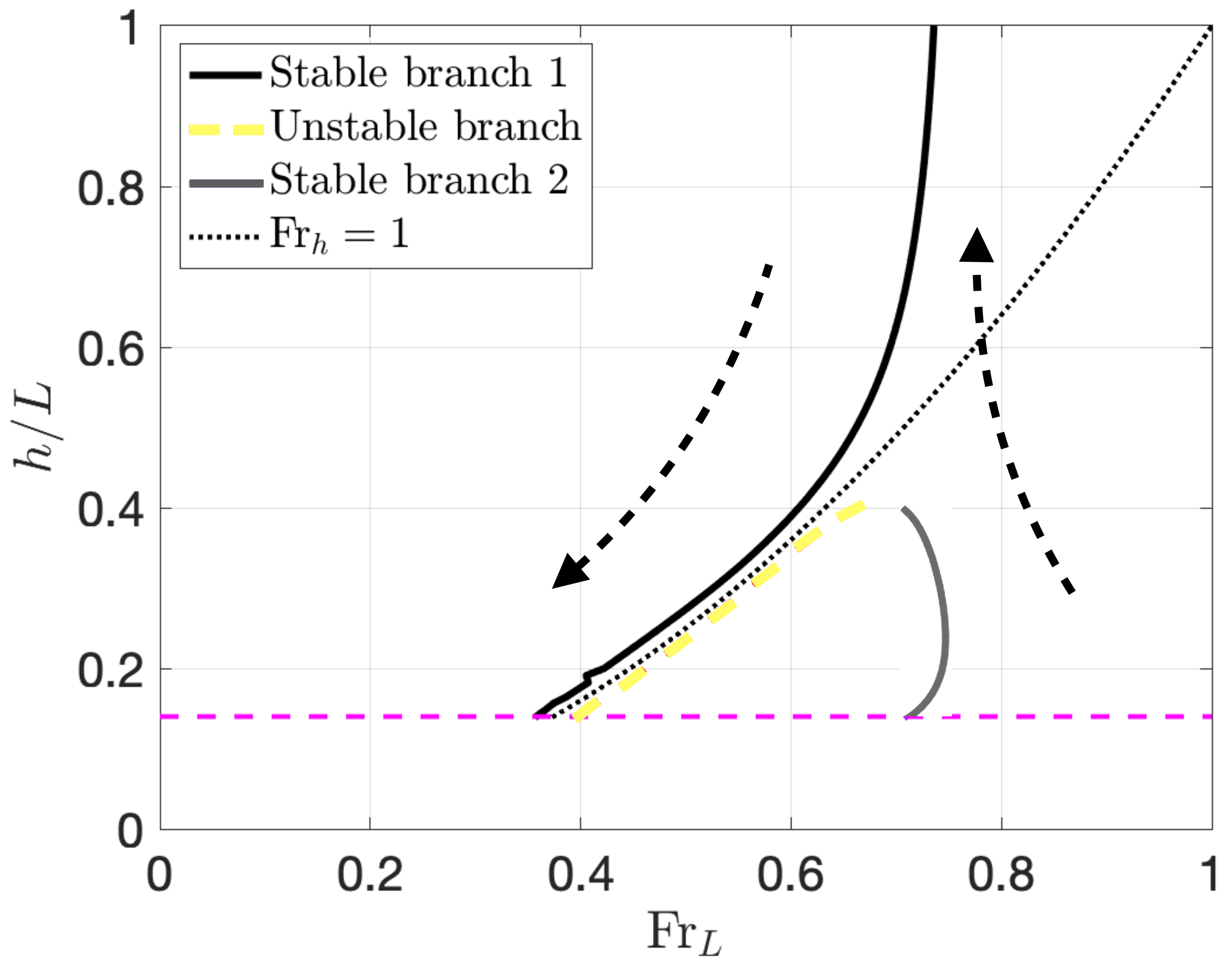
$$R = \mathrm{Drag}/\rho g L^3$$

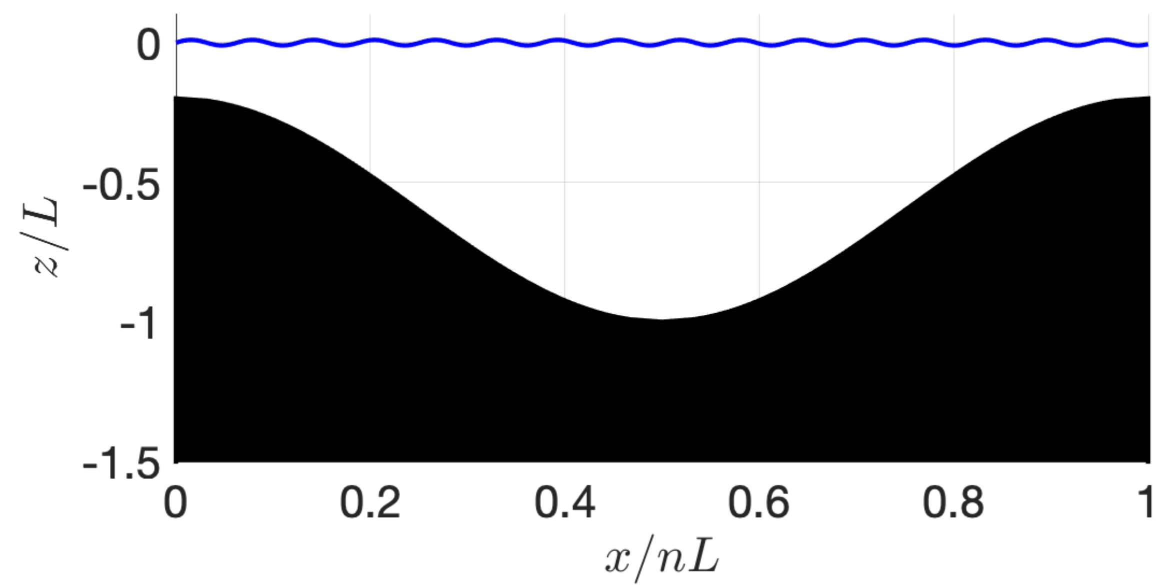


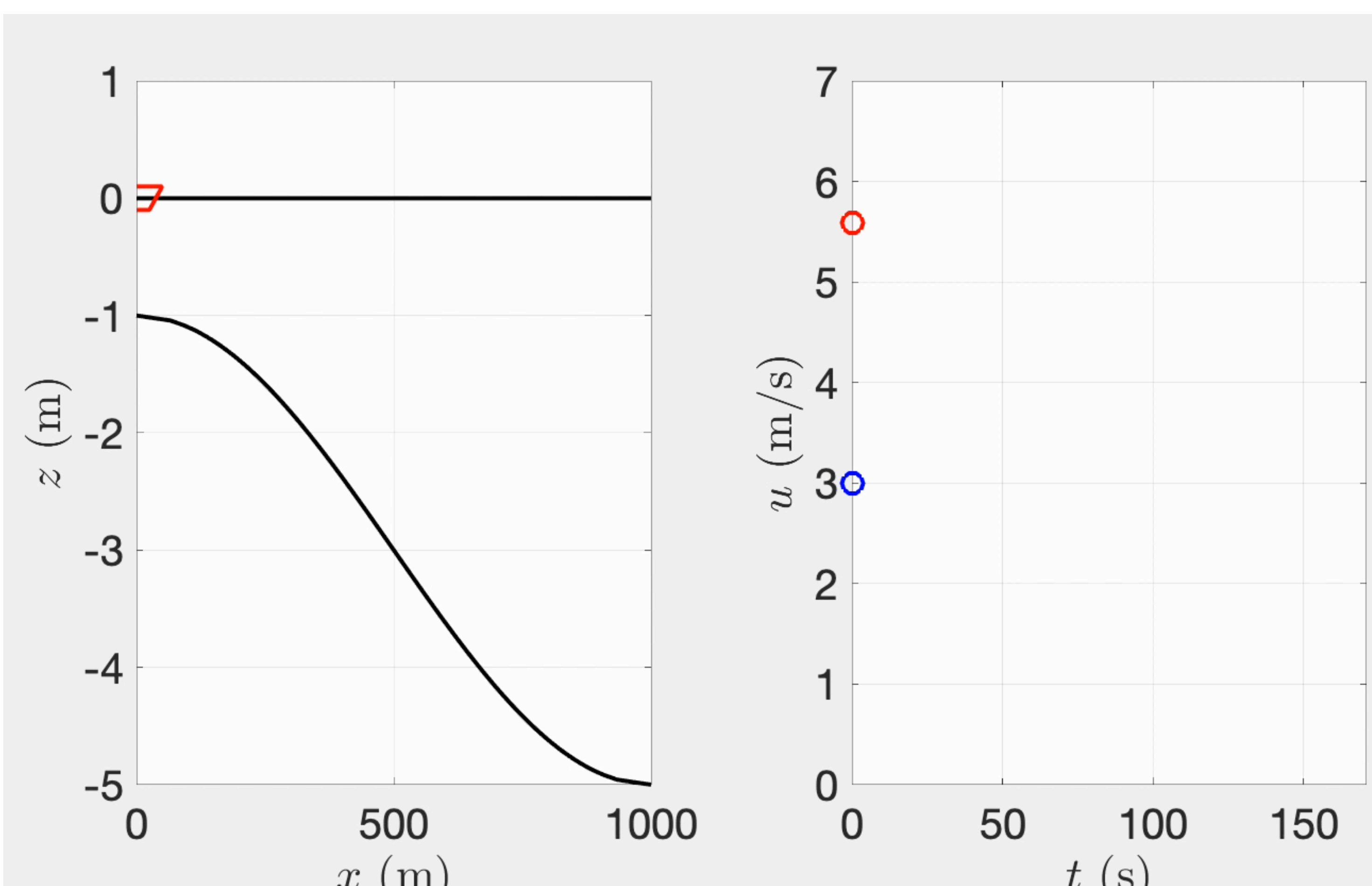

# ROWING RACE WITH NON-UNIFORM WATER DEPTH



# ROWING RACE WITH NON-UNIFORM WATER DEPTH

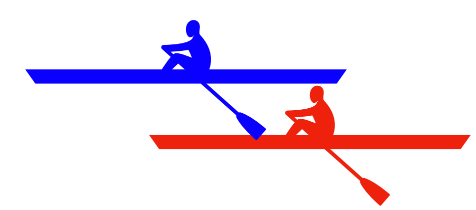



Pushed with *exactly* the same forcing!

Slow branch
Fast branch



Race times can be improved by a few %

*GPB, et al., PRF (2020)*

# WAVE THRUST PHENOMENA

*Jerome Neufeld*

*Muldrew Lake, Ontario, Canada (August, 2021)*

## “GUNWALE BOBBING”

# OLYMPIC CANOE SPRINT


Rio 2016 men's singles canoe 1000m


Amplitude: $A = 0.3\,\mathrm{m}$

Frequency: $\nu \approx 0.5\,\mathrm{Hz}$

Torso mass: $m \approx 20\,\mathrm{kg}$

Vertical force: $F_V = m\ddot{z} = mA(2\pi\nu)^2 \approx 60\,\mathrm{N}$

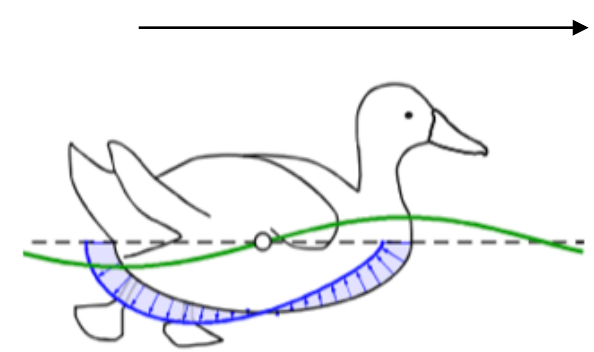
Horizontal force: $F_H \approx 300\,\mathrm{N}$

20% of the force injected vertically!

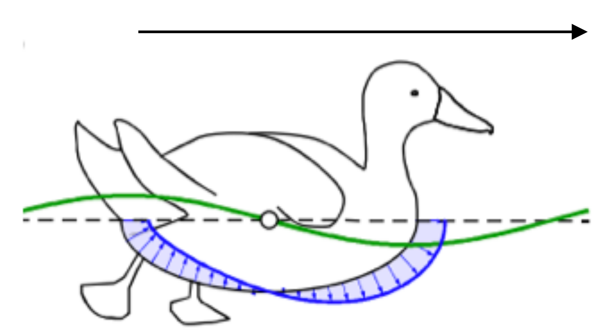
What is the impact on performance?

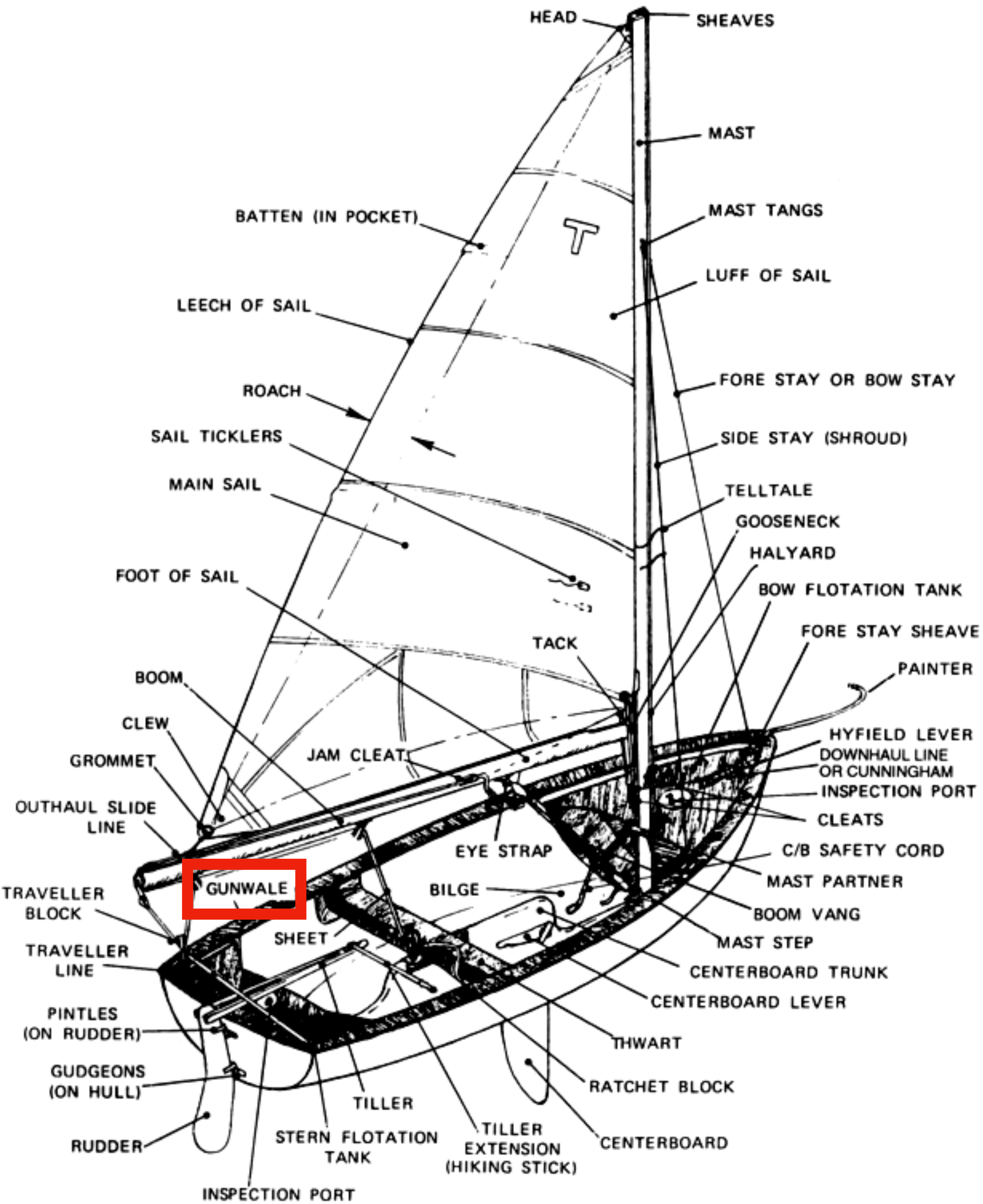
Can it be optimised?

**Against** surface gradients?



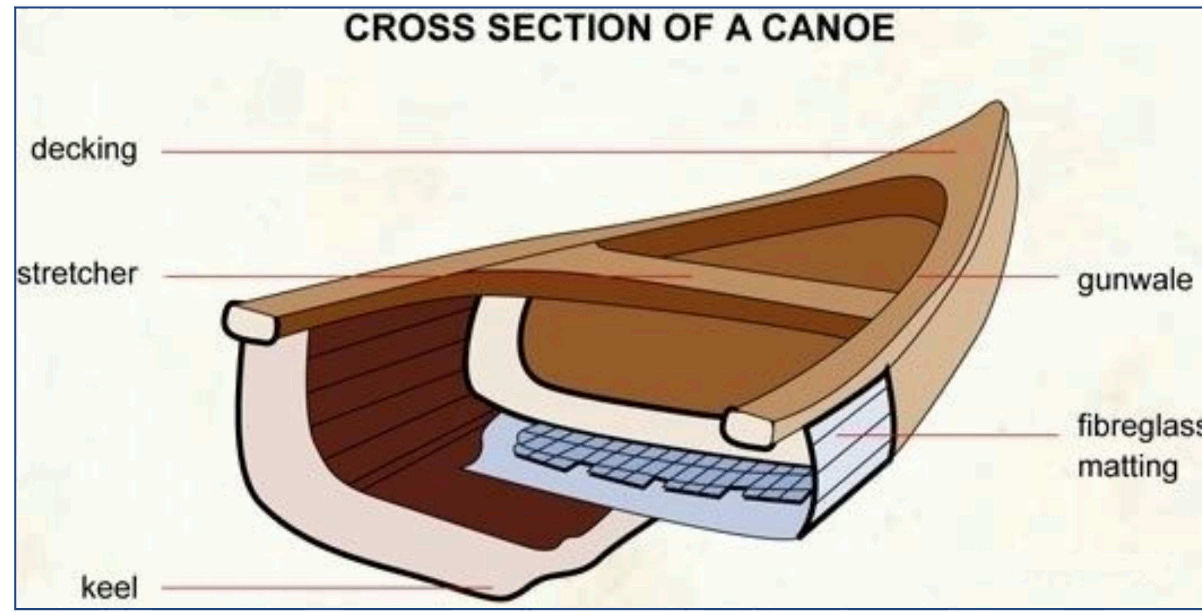
**With** surface gradients?






# GUNWALE BOBBING

*(pronounced gunnel)*



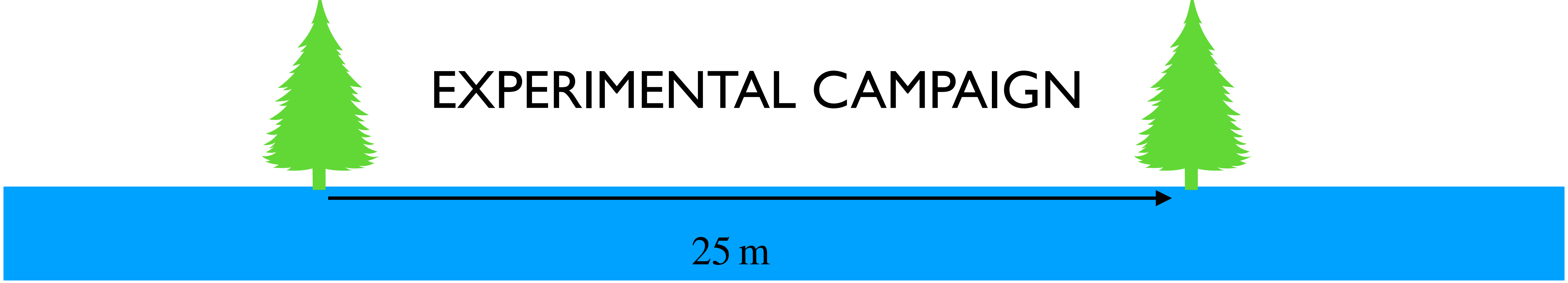

## gunwale

/ˈɡʌn(ə)l/

*noun*

the upper edge or planking of the side of a boat or ship.

# EXPERIMENTAL CAMPAIGN



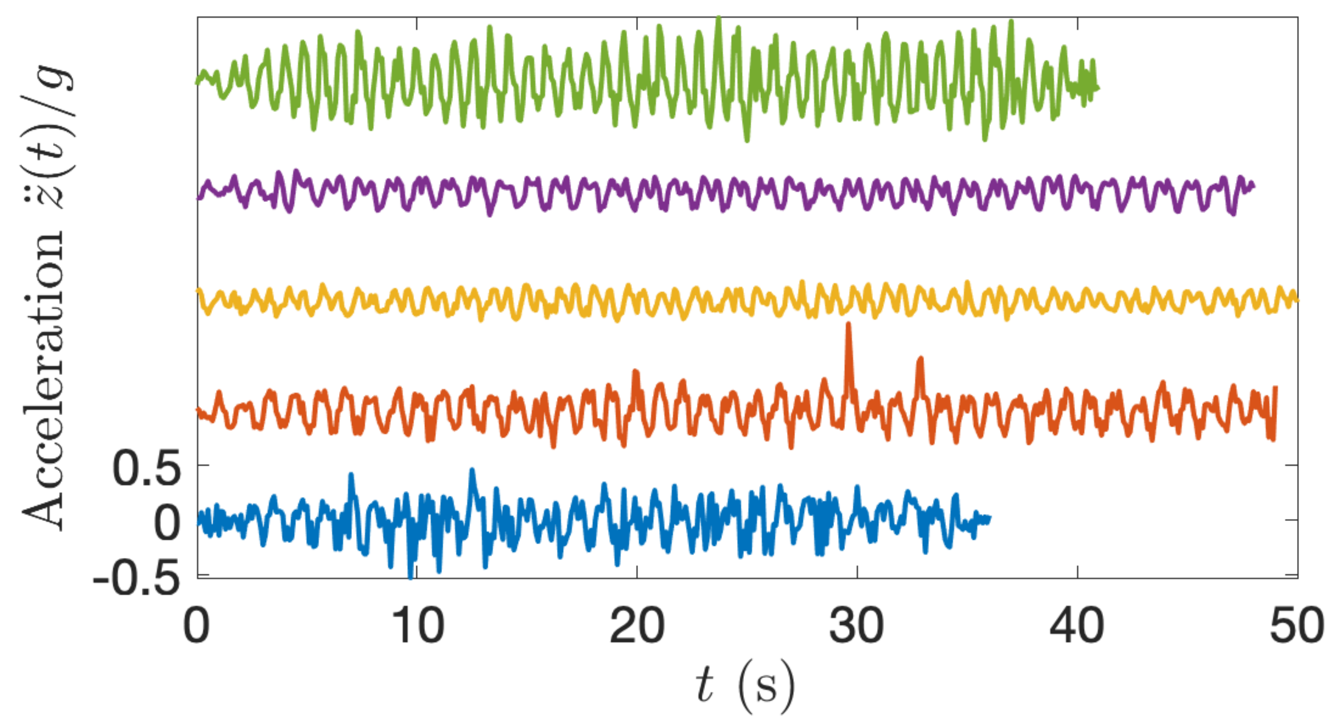


Ontario, Canada

Speed: $U = 0.5 - 1.25\,\mathrm{m/s}$

Frequency: $\nu = 1 - 1.5\,\mathrm{Hz}$

Accelerometer data




Paddle-board: $L = 3.05\,\mathrm{m}$

Canoe: $L = 4.70\,\mathrm{m}$

## DEEP WATER WAVES

Wavenumber: $k = \omega^2 / g$

Phase velocity: $c_p = \dfrac{\omega}{k} = \dfrac{g}{\omega}$

## DIMENSIONLESS PARAMETERS

Cruising Froude number: $\mathrm{Fr} = \dfrac{U}{(gL)^{1/2}} \approx 0.14$

Oscillating Froude number: $\mathrm{Fr}_\omega = \dfrac{c_p}{(gL)^{1/2}} \approx 0.19$

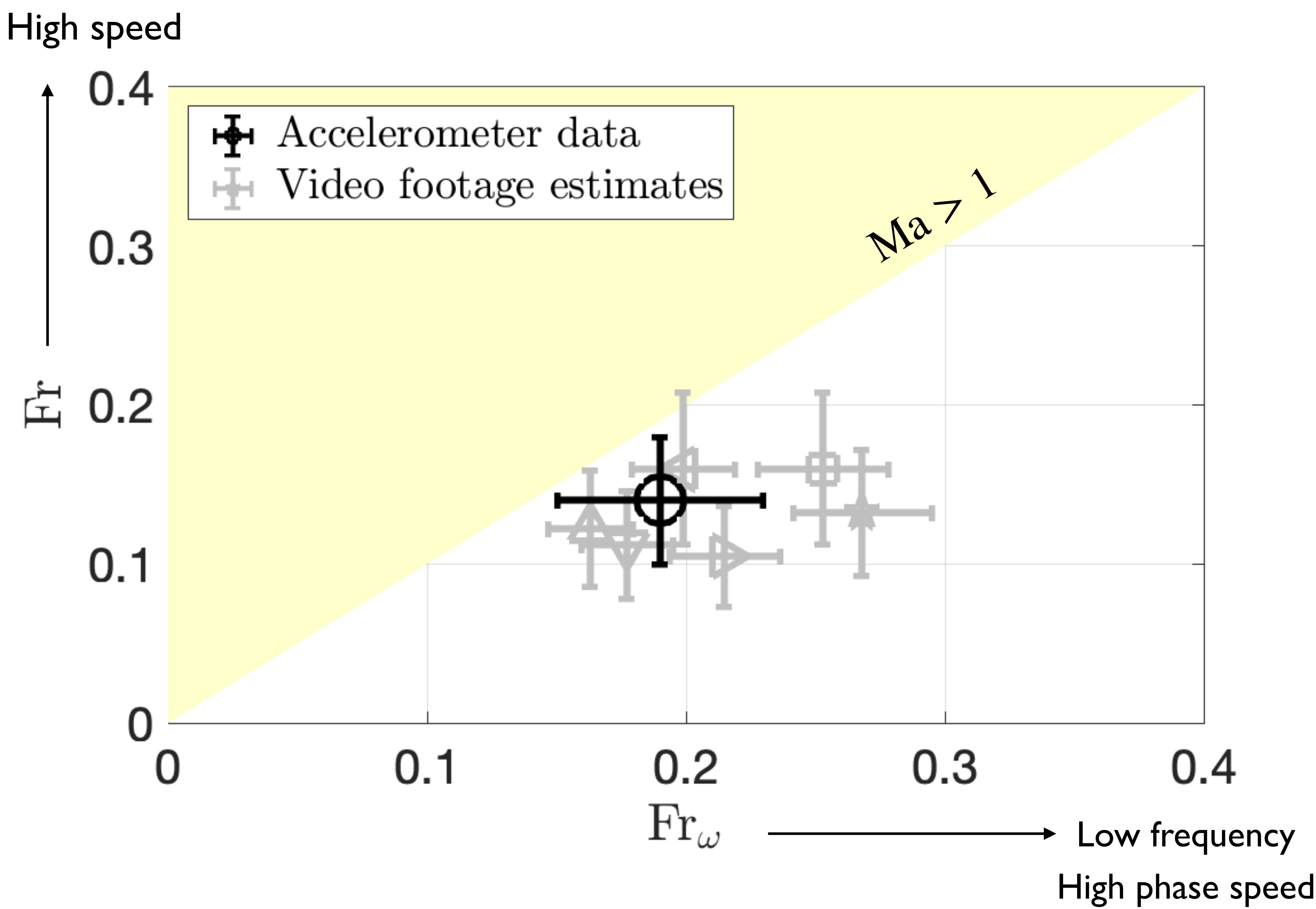
Mach number: $M = \dfrac{\mathrm{Fr}}{\mathrm{Fr}_\omega} = \dfrac{U}{c_p} \approx 0.73$

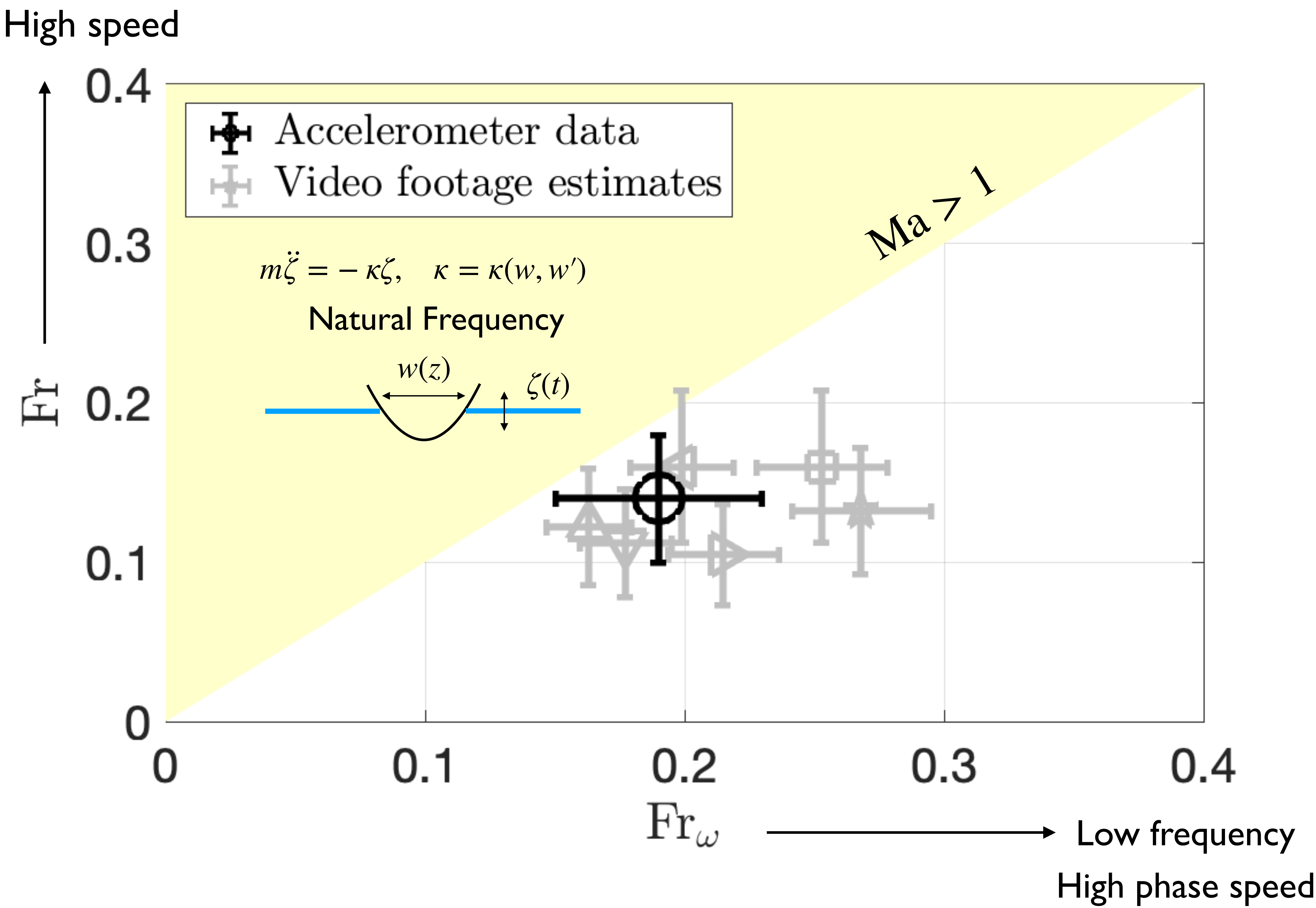
Miles Neufeld, Paddleboard, Canada

Martin Digby, Canoe, YouTube

GPB, Punt, Oxford

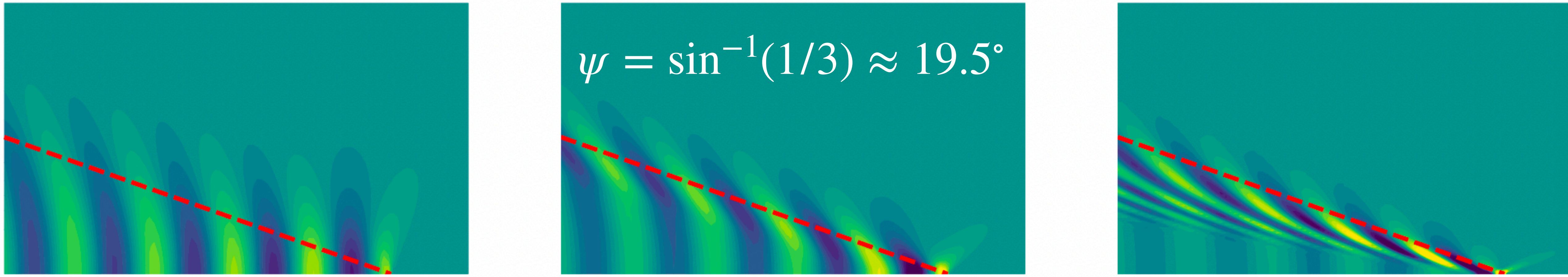
Jerome Neufeld, Canoe, Trinity College Cambridge


High speed
Accelerometer data
Video footage estimates
Ma > 1
Fr
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
$Fr_\omega$
Low frequency
High phase speed


High speed
Accelerometer data
Video footage estimates
Ma > 1
$m\ddot{\zeta} = -\kappa\zeta, \quad \kappa = \kappa(w, w')$
Natural Frequency
$w(z)$
$\zeta(t)$
Fr
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
$\mathrm{Fr}_\omega$
Low frequency
High phase speed

## KELVIN ANGLE

Fr=0.3 Fr=0.5 Fr=1

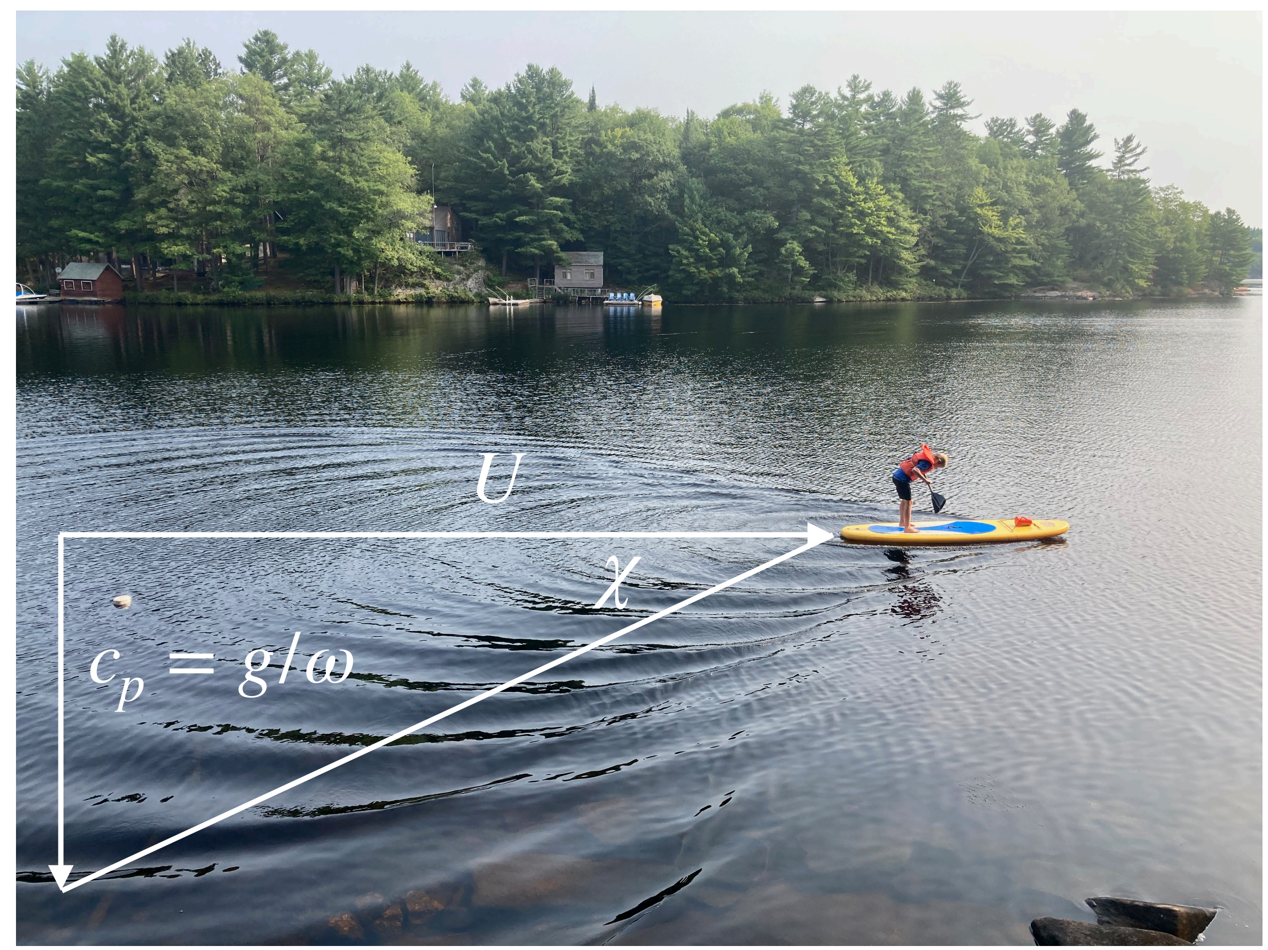



## MACH ANGLE

$$\chi = \tan^{-1}(g/U\omega)$$

$$= \tan^{-1} 1/\mathrm{Ma}$$

Waves due to bouncing (Mach) are **dominant** compared with waves due to cruising (Kelvin)

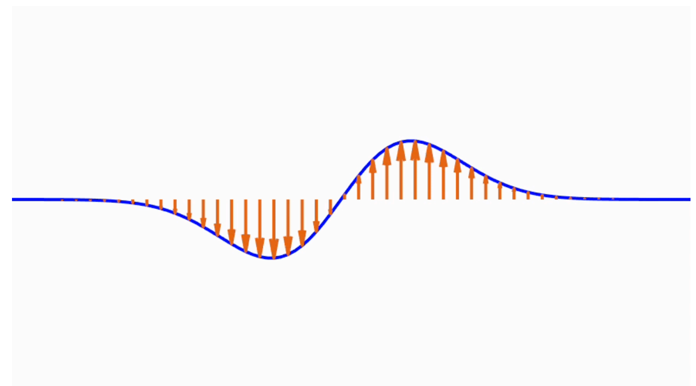


# LINEAR MODEL FOR GUNWALE BOBBING

Heaving $p_H$

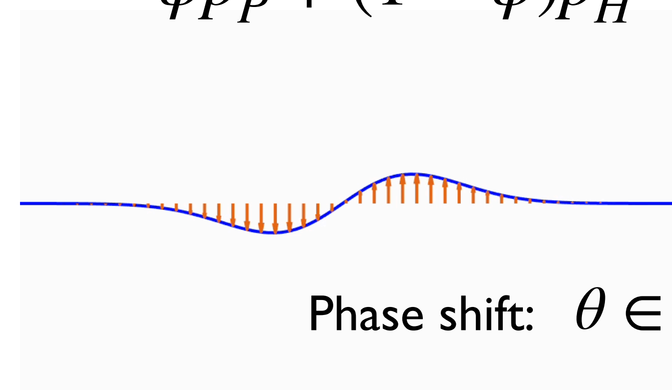


Pitching $p_P$



Heaving and pitching

$\phi p_P + (1 - \phi) p_H$



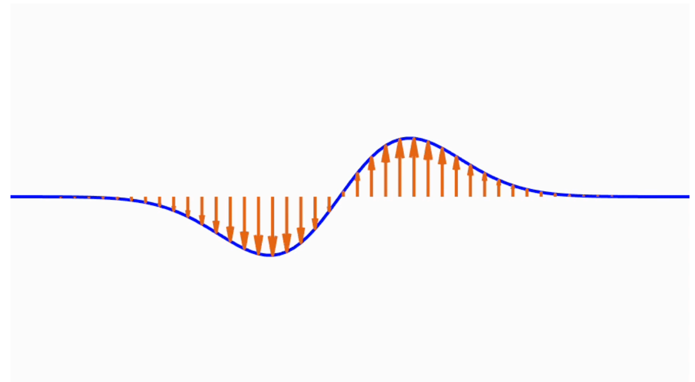
Phase shift: $\theta \in [0, \pi]$

# LINEAR MODEL FOR GUNWALE BOBBING

Heaving $p_H$

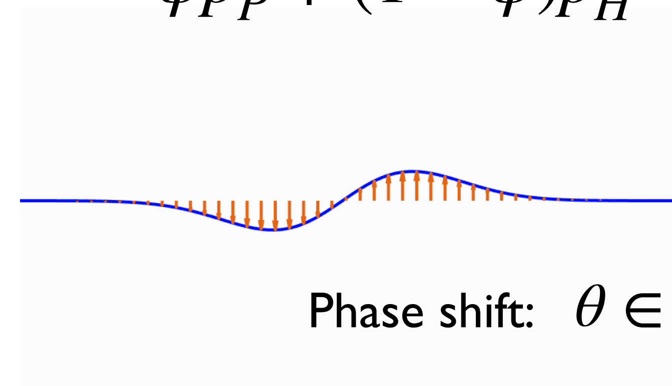


Pitching $p_P$



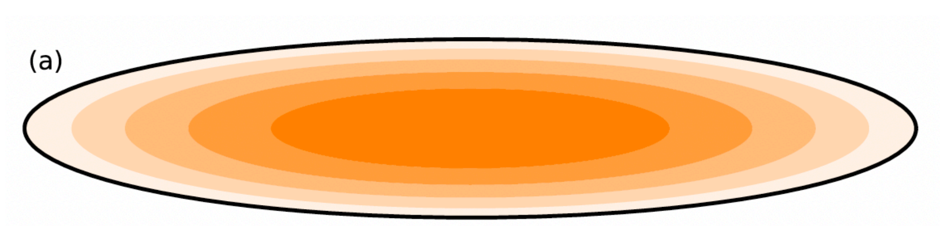
Heaving and pitching

$\phi p_P + (1 - \phi) p_H$

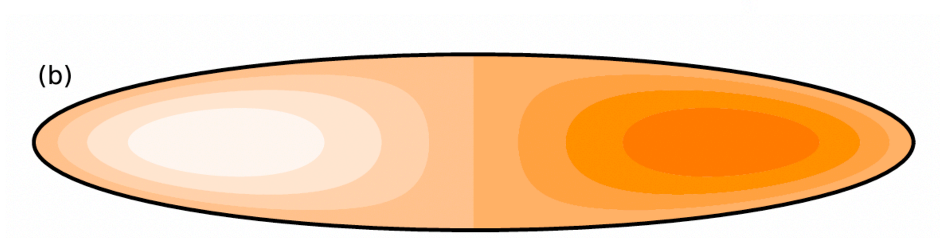


Phase shift: $\theta \in [0, \pi]$

$$p_H = \mathrm{Im}\left\{p_C(X, y) e^{i\omega t}\right\}$$




$$p_P = \mathrm{Im}\left\{p_C(X, y) \frac{X}{L} e^{i(\omega t + \theta)}\right\}$$




Canoe shape: $f = e^{-x^2/2L^2 - y^2/2W^2}$

Cruising: $p_C = \rho g D f(X, y)$

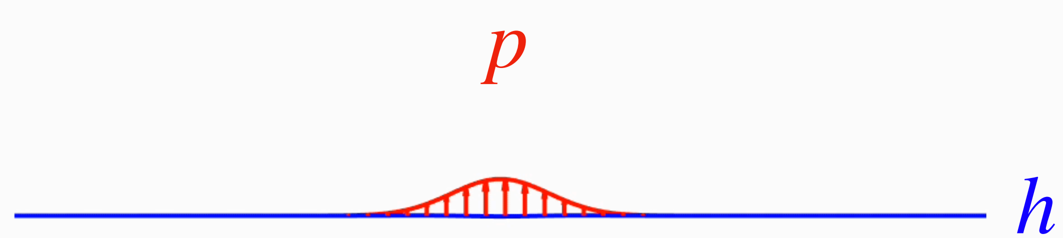

$X = x - Ut$


Dimensionless parameters:

$$\alpha = L/W, \quad \beta = L/D, \quad \phi, \quad \theta, \quad \mathrm{Fr}_\omega, \quad \mathrm{Fr}$$

Minimal (non-dispersive) model

$$\frac{1}{c_p^2}\frac{\partial^2 h}{\partial t^2} + \nabla^2 h = \frac{1}{\rho g}\nabla^2 p$$



1. Apply a Lorentz transformation:

$$\gamma = (1 - U^2/c_p^2)^{-1/2}$$

$$\tilde{X} = \gamma(x - Ut)$$

$$\tilde{y} = y$$

$$\tilde{t} = \gamma(t - Ux/c_p^2)$$

1. Apply a Lorentz transformation:

$$\tilde{X} = \gamma(x - Ut)$$

$$\tilde{y} = y$$

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2. Seek a solution:

$$h = \mathrm{Im}\left\{\bar{h}e^{i\omega t}\right\}$$

Helmholtz Equation

$$(\tilde{\nabla}^2\bar{h} + k^2\bar{h} = -\tilde{\nabla}^2\bar{p}/\rho g)$$

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$$\tilde{X} = \gamma(x - Ut)$$
$$\tilde{y} = y$$
$$\tilde{t} = \gamma(t - Ux/c_p^2)$$

$$\gamma = (1 - U^2/c_p^2)^{-1/2}$$


2. Seek a solution:

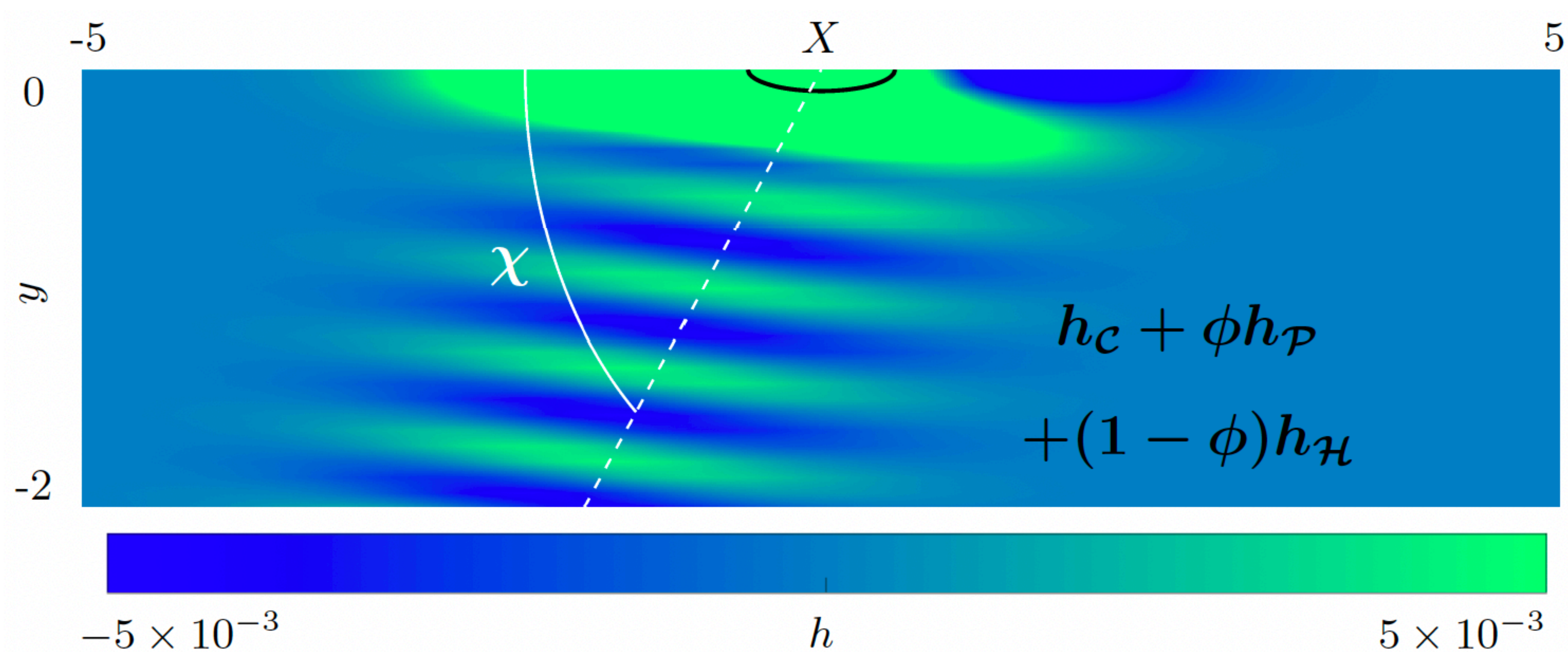
$$h = \mathrm{Im}\left\{\bar{h}e^{i\omega t}\right\}$$

Helmholtz Equation

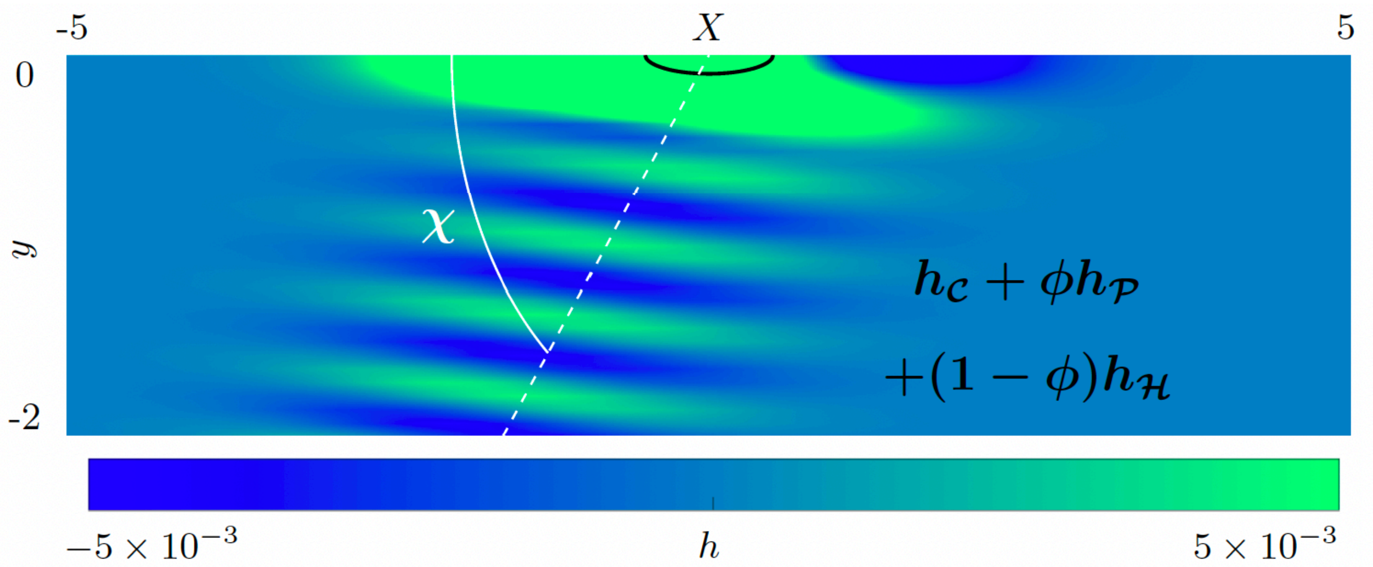
$$(\tilde{\nabla}^2\bar{h} + k^2\bar{h} = -\tilde{\nabla}^2\bar{p}/\rho g)$$

3. Method of Green's functions:

$$\bar{h} = \iint_{-\infty}^{+\infty} H_0^{(1)}(|\tilde{\mathbf{X}} - \tilde{\mathscr{X}}|)\frac{\tilde{\nabla}^2\bar{p}(\tilde{\mathscr{X}})}{\rho g}\,\mathrm{d}\tilde{\mathscr{X}}$$



# COMPARISON WITH DATA




## THRUST-DRAG BALANCE

$$F = \iint p \frac{\partial h}{\partial X} \, \mathrm{d}S = \frac{1}{2} \rho C_D U^2 A$$

$$\implies \quad U \approx 1.5 \, \mathrm{m/s}$$

## WAKE ANGLE

$$\chi = \tan^{-1} 1/\mathrm{Ma} \approx 51^{\circ}$$

## OTHER FINDINGS:

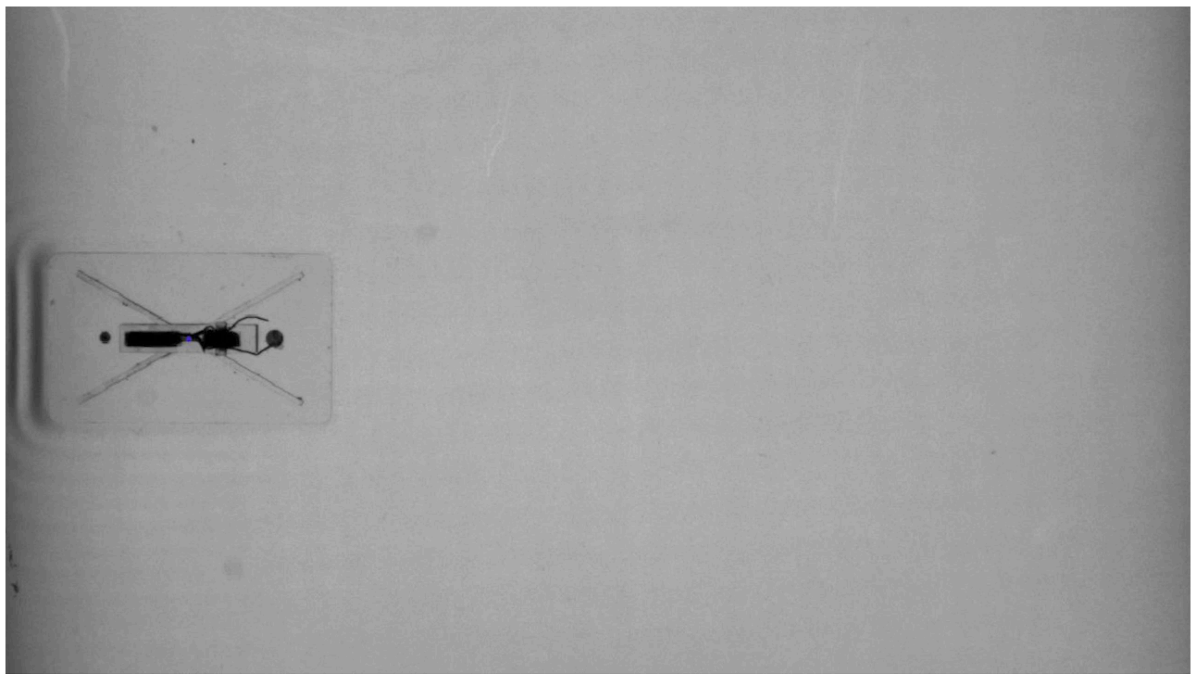
Must have *heaving* ***and*** *pitching* to generate thrust

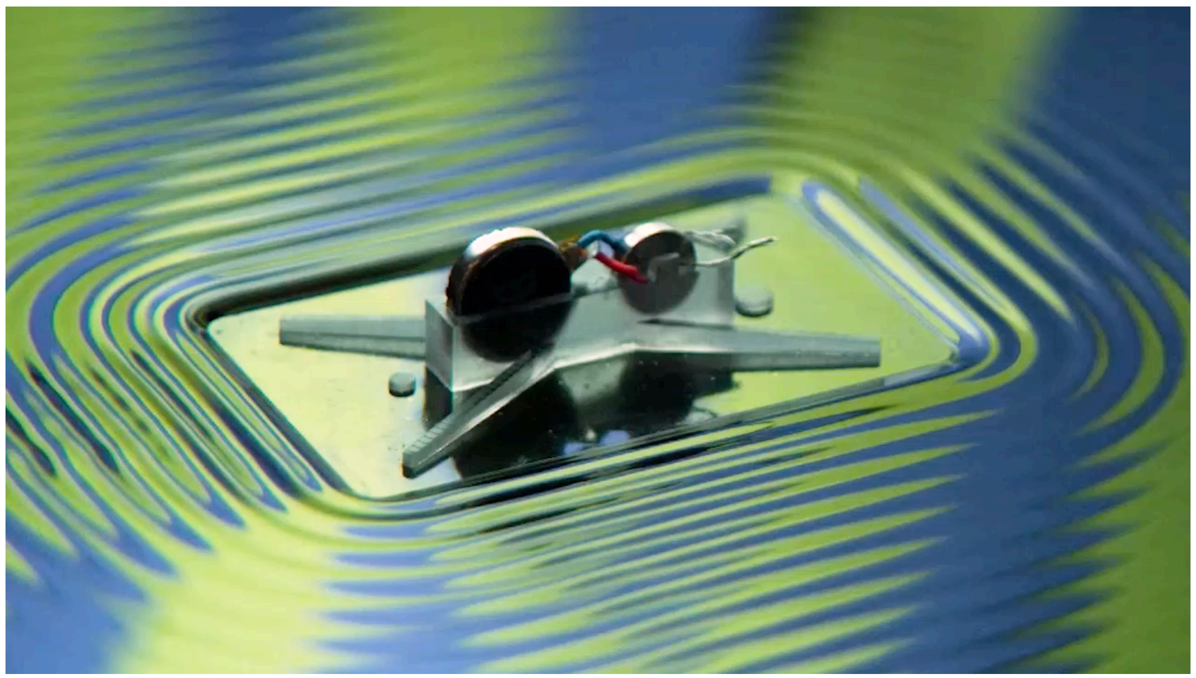
*Optimum parameters:* $\phi = 1/2, \quad \theta = \pi/2$

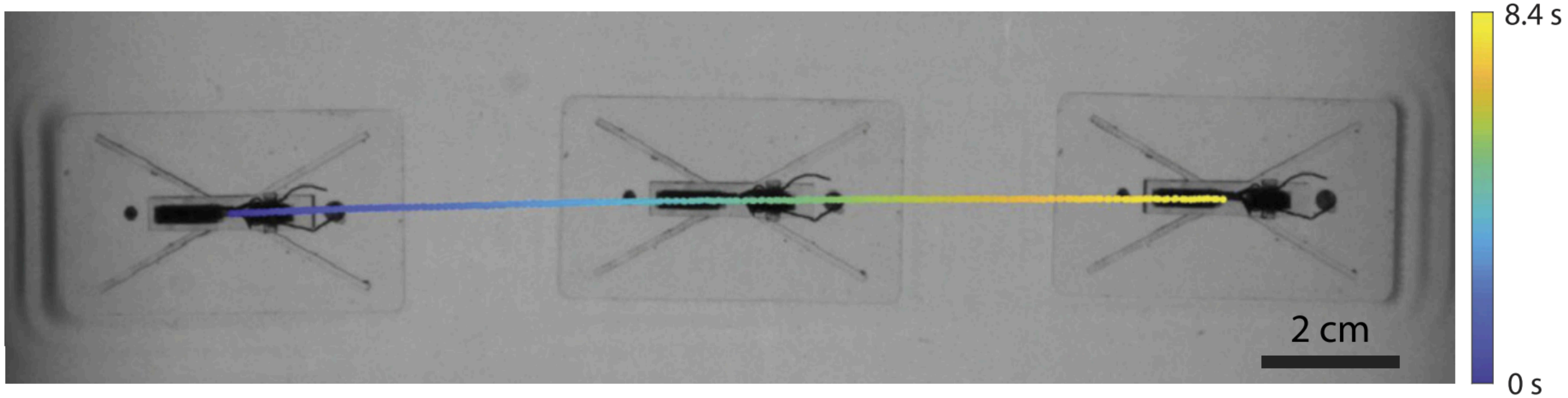
*GPB, et al, PRF (2022) + Nature (Research Highlight)*

# SURFERBOT

## OSCILLATING, FLOATING RAFT WITH ASYMMETRIC WEIGHT








Gunwale bobbing at small scale

*Rhee, Hunt, Thomson & Harris. (2022)*

# LINEAR DISPERSIVE MODEL
# FOR WAVE-DRIVEN PROPULSION

KINEMATIC + DYNAMIC BC

$$\phi_z = \dot{\zeta} + x\dot{\theta}$$

$$\phi_z = \frac{\phi_{tt}}{g} - \frac{\gamma\phi_{zxx}}{\rho g} + \frac{4\nu\phi_{txx}}{g}$$

OSCILLATING RAFT

$x \to -\infty$

RADIATIVE BC

$$\nabla^2\phi = 0$$

$x \to \infty$

RADIATIVE BC

$\phi_z \to 0$ $\quad z \to -\infty$

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$x \to \infty$

RADIATIVE BC

$\phi_z \to 0$ $\quad z \to -\infty$

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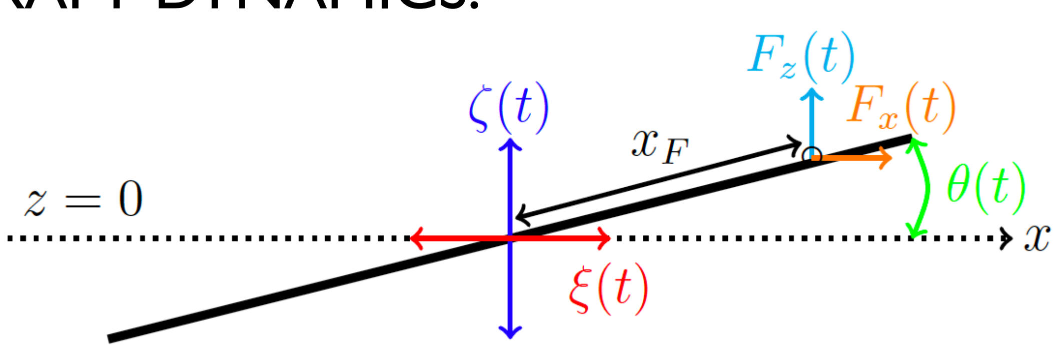
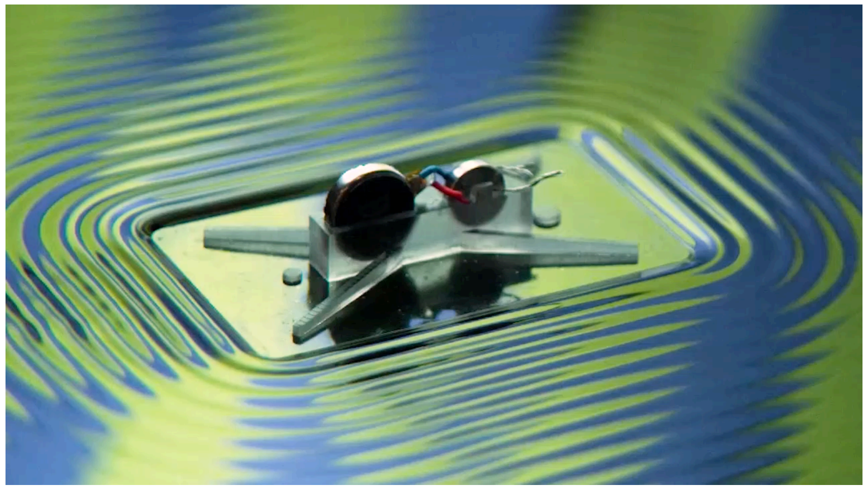
… + RAFT DYNAMICS:



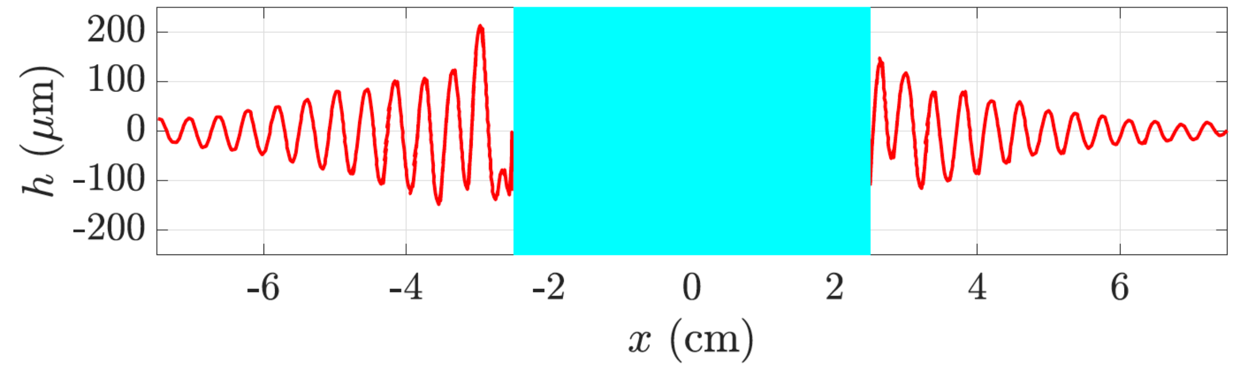

$$m\ddot{\mathbf{X}} = \mathbf{F} + \int_S (p - p_a)\mathbf{n}\,\mathrm{d}x - \mathbf{F}_d + m\mathbf{g},$$

$$I\ddot{\boldsymbol{\theta}} = \mathbf{x}_F \times \mathbf{F} + \int_S (p - p_a)\mathbf{x} \times \mathbf{n}\,\mathrm{d}x,$$

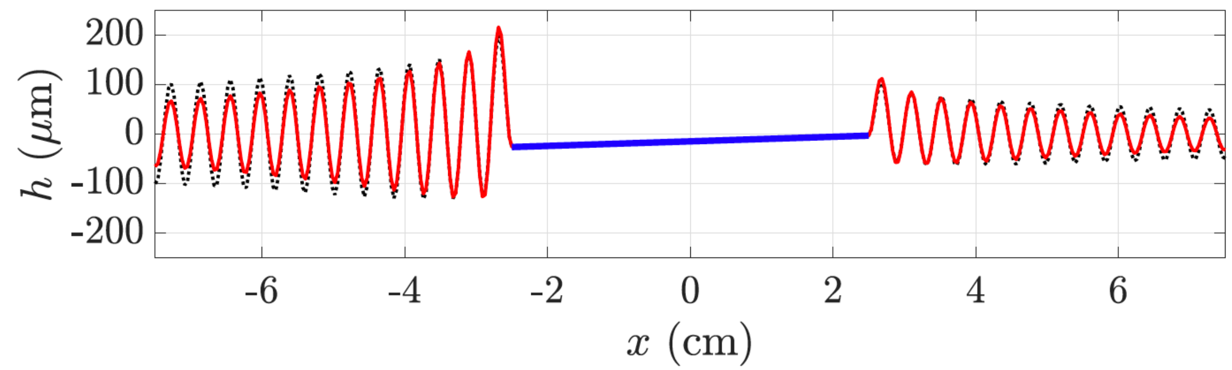
# SURFERBOT DATA



# WAVE HEIGHT



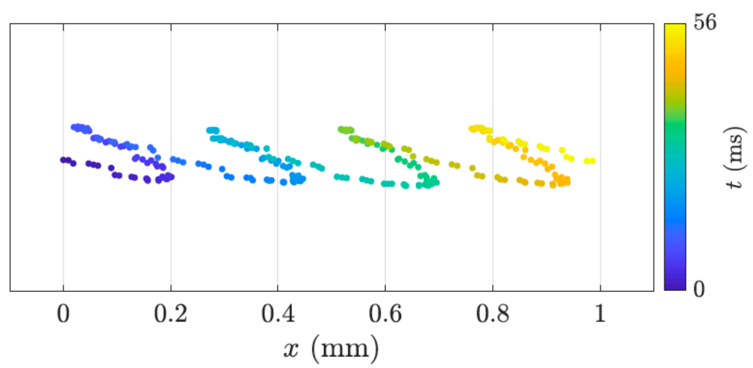

EXPERIMENT

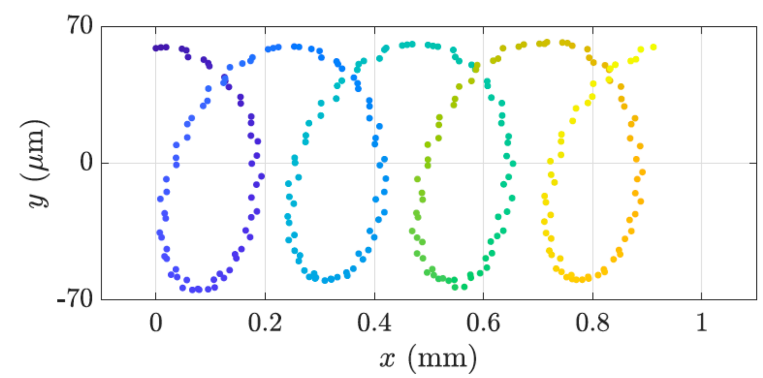



THEORY

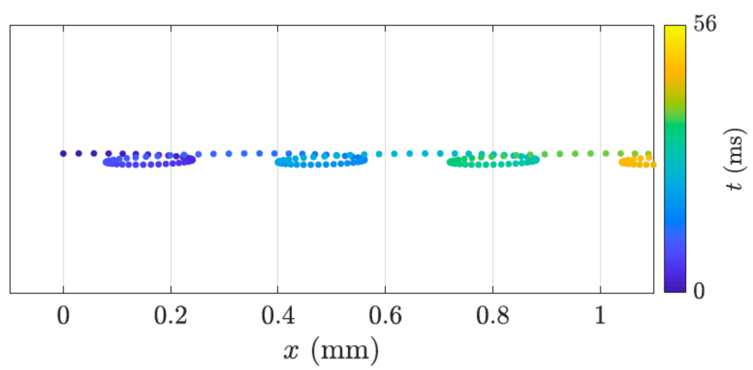
# FRONT POSITION

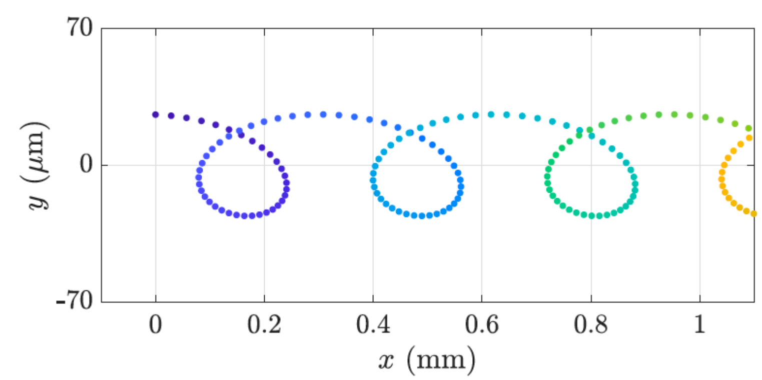
# REAR POSITION



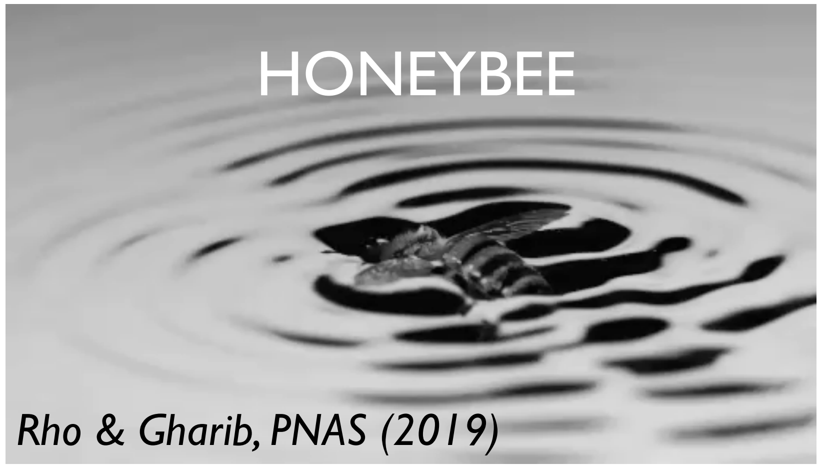

EXPERIMENT



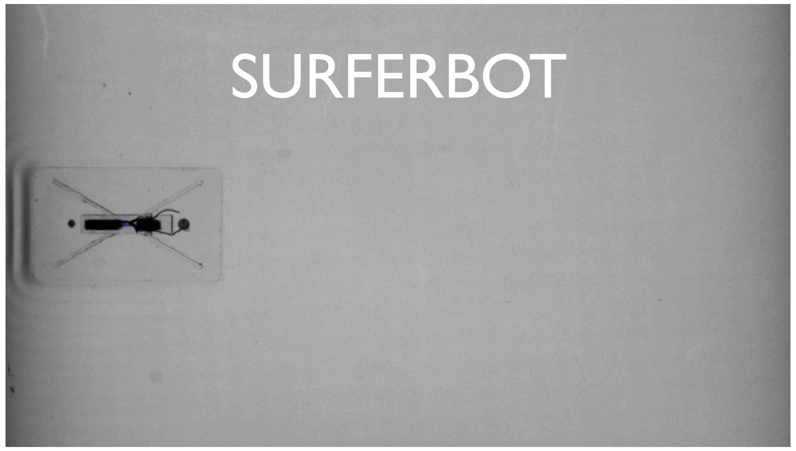

THEORY

# WAVE-DRIVEN PROPULSION ACROSS SCALES



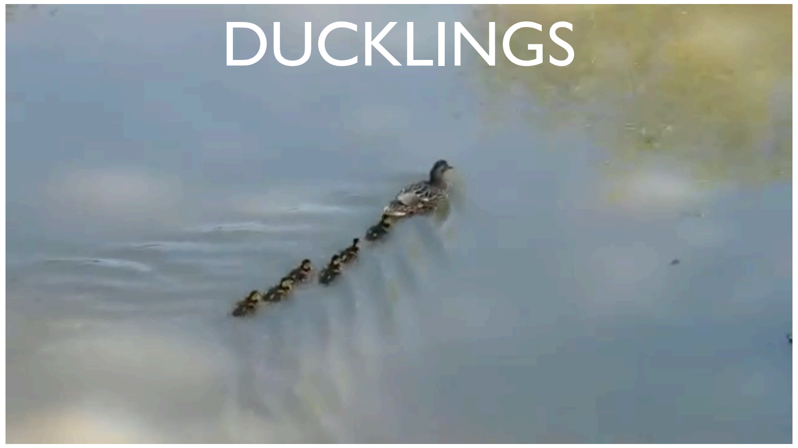

$\mathrm{Fr} = 0.1$ $\mathrm{Fr}_\omega = 0.07$

$\mathrm{Re} = 10^2$ $\mathrm{We} = 0.1$




$\mathrm{Fr} = 0.03$ $\mathrm{Fr}_\omega = 0.03$

$\mathrm{Re} = 10^3$ $\mathrm{We} = 0.3$




$\mathrm{Fr} = 0.7$

$\mathrm{Re} = 10^4$ $\mathrm{We} = 10^2$

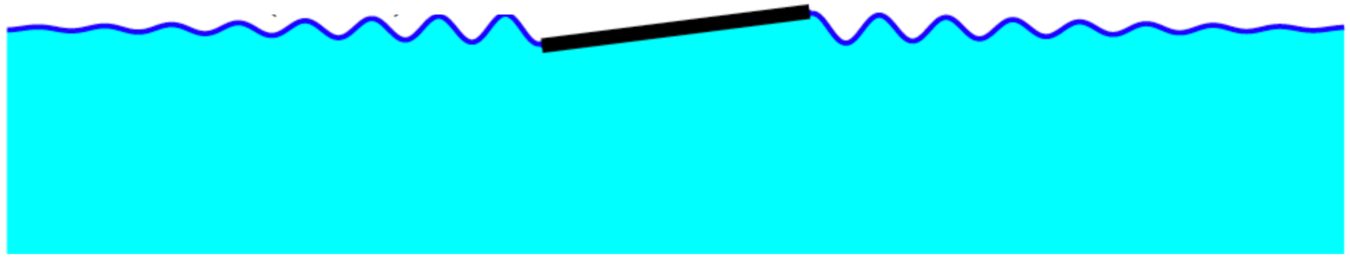

$\mathrm{Fr} = 0.2$ $\mathrm{Fr}_\omega = 0.2$

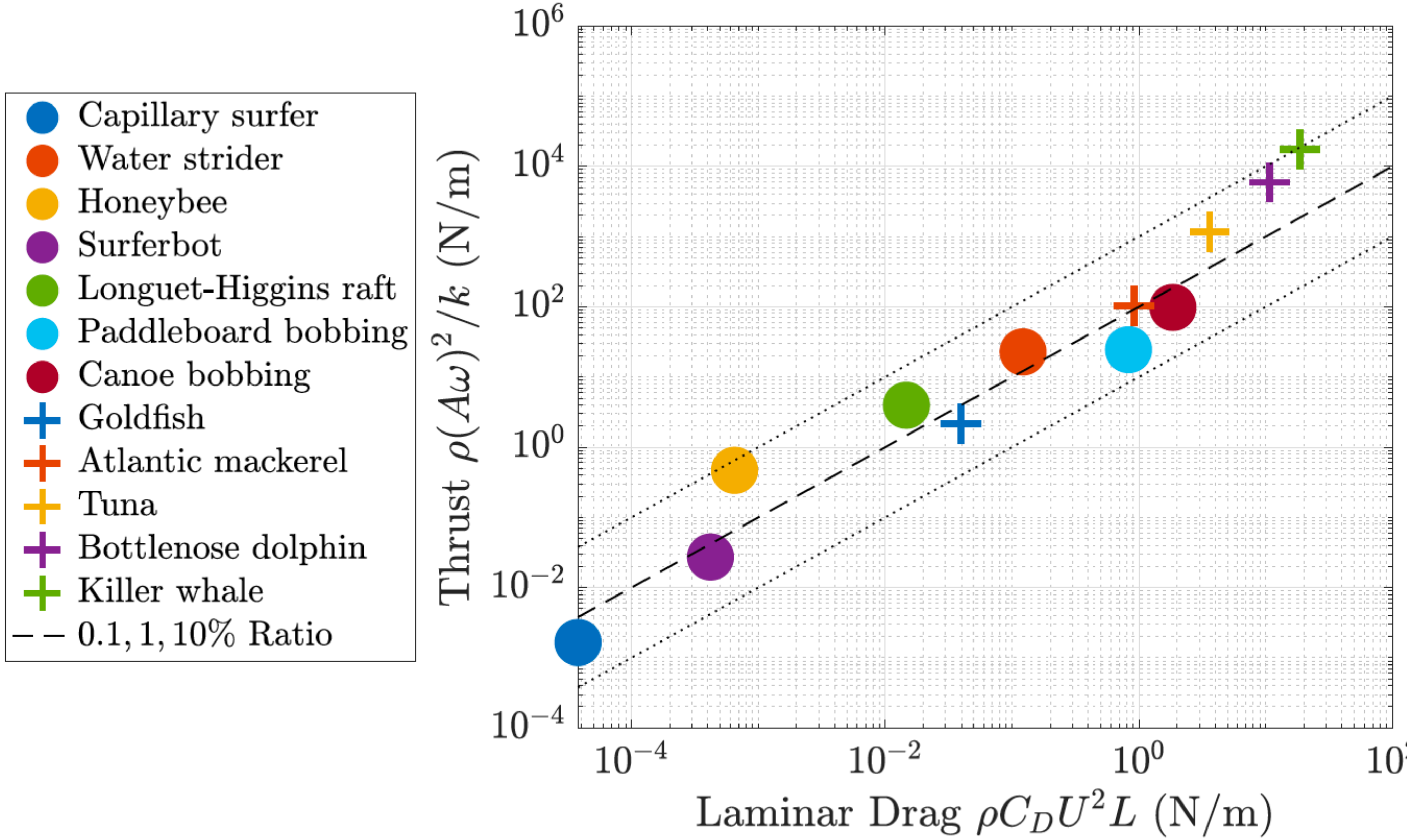
$\mathrm{Re} = 10^6$ $\mathrm{We} = 10^5$

$\mathrm{Re} = UL/\nu$ $\mathrm{Fr} = U/\sqrt{gL}$ $\mathrm{Fr}_\omega = \omega^{-1}\sqrt{g/L}$ $\mathrm{We} = \rho U^2 L/\gamma$

THRUST FROM OSCILLATIONS:

$$\sim \frac{\rho(A\omega)^2}{k}$$






*GPB, Devauchelle & Thomson (in preparation)*


QUESTIONS?